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The Boundary Mapped Collocation Method for Heat Conduction Problems with Heat Generation Spatially Varying Conductivity

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Abstract. In this paper, the boundary mapped collocation (BMC) approach is presented for the analysis of heat conduction problems involving heat generation and non-homogeneous thermal conductivity. The proposed methodology is introduced to produce the numerical solutions of the temperature field within the framework of the BMC method, a novel boundary meshless method, without resorting to requiring any integral calculation, neither in the domain nor at the boundary. In particular, the arrangement of discrete nodes is restricted to the axis, which brings the spatial dimension down by one. The technique also reduced the traditional complex shape functions to succinct one-dimensional boundary shape functions by using one-dimensional basis functions and weight functions for two- and three-dimensional approximation implementation. In addition, four numerical applications and comparisons with the outcomes of the finite element approach and another meshfree method are used to demonstrate the correctness, convergence, and stability of the BMC method.

AMS subject classifications: 80M22, 65-04, 68-04

Key words: Boundary mapped collocation method, heat conduction, heat generation, spatially varying conductivity, meshless method.

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1 Introduction

One of the most significant jobs for the scientific and industrial communities has always been the thorough investigation of heat transport issues through the solution of various partial differential equations in numerous engineering domains. Particularly, the heat conduction issues of functionally graded materials (FGM) with anisotropic mechanical properties have been receiving increasing attention for decades. However, analytical solutions for these heat conduction issues are hardly ever developed because of the variability of the material. As a result, computer-based techniques have been widely used in the study of heat conduction. The entire discretization of the computation domain is not necessary when using domain discretization techniques like the finite difference method (FDM) [1], finite volume method (FVM) [1], and finite element method (FEM) [2, 3] to solve these problems. The fundamental drawback of domain discretization techniques is the complexity of shape mesh production, which is computationally expensive. In addition, the boundary element method (BEM) [4–8] has been proposed as an effective alternative to the discretization of the boundary over the boundary.

Although the aforementioned methods have been applied to conduct heat transfer analysis successfully, they inherently own several drawbacks. The FEM, as one of the most popular numerical methods, resorts to the generation mesh using a polynomial representation for approximation of the solution region. Nevertheless, due to the C^0 continuous of the FEM shape function [9, 10], the mesh generation not only is time-consuming but also leads to the error with complex geometrical shape discretization. Also, since domain integrals are included in the integral equations, the main challenges faced by the BEM are the costly numerical integrations and computational expenses for an irregular region meshing [11, 12]. To circumvent the disadvantage of mesh generation, an alternative referred to as the meshfree method, such as large deformation, high gradient, and discontinuities.

In recent years, numerous meshless methods have been proposed to promote the achievement of approximate solutions for heat conduction analyses systematically. Sigh et al. [13] introduced the element-free Galerkin (EFG) method to analyze transient nonlinear thermal conduction within a solid structure. Zhang et al. [14] improved the EFG method for analyzing isotropic thermal conductivity. Gao [11] proposed the meshless BEM to analyze heat conduction with anisotropic thermal conductivity. Cui et al. [15] proposed the radial integration polygonal boundary element method (RIPBEM) for thermal conduction analyses with complex 3D geometries. Wu and Tao [16] employed the meshless local Petrov-Galerkin (MLPG) method to analyze steady-state thermal conduction with complicated boundary conditions. Li et al. [17] analyzed thermal conduction by combining the MLPG and FVM methods within two-dimensional irregular geometry. Hidayat [16] presented a meshless finite difference approach with B-splines technique for solving coupled advection-diffusion-reaction problems.

However, since these weak-form methods could not get rid of a grid generation with numerical integration, the above EFG, BEM, and MLPG methods are not complete mesh-