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Unified Solution of Conjugate Fluid and Solid Heat Transfer–Part II. High-Order Conjugate Heat Transfer

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Abstract. A high-order unified solution method is presented for simulating the coupled fluid flow and heat transfer phenomena in solid materials. The method integrates the energy transfer processes of fluid and solid by transforming the compressible Navier-Stokes equations into a dimensionless system through a double-time-scale approach. A time scaling factor is introduced into the system to bridge and expedite the energy transfer processes between the mediums, thereby accelerating convergence towards a global steady state. To ensure consistent accuracy across material interfaces, high-order formulations are introduced to obtain the interface temperature and heat flux. The effectiveness of the methods are verified and validated through numerical experiments with a three-dimensional discontinuous Galerkin flow solver. Numerical results demonstrate the globally high-order accuracy of the unified method for fluid-solid conjugate heat transfer problems, enabling rapid and robust convergence with large CFL numbers of up to 10^8.

AMS subject classifications: 65M22, 76N06

Key words: Conjugate heat transfer, unified solution, Navier Stokes, high order, discontinuous Galerkin.

1 Introduction

The coupling of fluid flow and heat transfer in solids presents numerous scientific and engineering challenges. Various applications necessitate addressing conjugate heat transfer (CHT) effects, including turbomachinery, heat exchangers, semiconductor devices, and nuclear reactors, among many others. Traditionally, CHT problems are modeled by combining the Navier-Stokes (N-S) equations for fluid flow with the Fourier-Biot (F-B) equation for solid heat transfer [7,21]. This approach usually utilizes separate solvers for each

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of the physical field with loose coupling through the exchange of boundary conditions across the domain interfaces [27]. However, loose coupling methods often lead to stability constraints on time step size [5,20,22], resulting in robustness issues and massive time steps. Alternatively, fully coupling methods try to combine the fluid and solid governing equations by incorporating interface conditions into a single system. Nonetheless, developing a fully coupling solver by combining the N-S and F-B equations can be as labor-intensive as developing separate codes for fluid and solid applications, since combing the different existing algorithms and solvers is challenging, if not impossible [10,18,24].

In conjugate heat transfer problems involving incompressible flows, two prevalent approaches were proposed to model the solid heat transfer phenomena. The first approach employs the Fourier-Biot (F-B) equation, while the second one utilize the energy equation of the incompressible Navier-Stokes (N-S) equations. The latter is viable since the incompressible energy equation can be decoupled from the continuity and momentum equations. However, for the compressible N-S equations, the situation is different since the energy equation is coupled with the continuity and momentum equations. Nordström et al [19] presented a similarity condition between the compressible N-S equations and the F-B equation, but the coupling method at the fluid-solid interface still imposes some limitations due to interface stability constraints. To alleviate these stability constraints, we propose a unified solution framework that can fully inherit the computational capabilities of an existing high-order compressible N-S flow solver. In Part I of this work [15], a new high-order solid heat transfer solver has been developed based on the compressible N-S equations. This paper serves as Part II of the work and extend the proposed method to conjugate fluid and solid heat transfer problems.

The remaining parts of this paper are organized as follows: Section 2 presents the theory and equations of the unified solution approach. Section 3 discusses the numerical schemes. Various CHT problems are tested in Section 4. Finally, concluding remarks are given in Section 5.

2 Governing equations

2.1 Nondimensionlization

The compressible N-S equations are employed to describe the coupled fluid flow and solid heat conduction in a unified solution framework. Consider a material with the mass density ρ, temperature T, pressure p, velocity $\mathbf{u}$, total energy E and total enthalpy H per unit mass, the governing equations containing the conservation of mass, momentum and energy, optionally with the gravitational acceleration vector $\mathbf{g}$ and the volumetric heat source Q, are written as: for $t>0$ and $\mathbf{x}\in\Omega_{\text{fluid}}\cup\Omega_{\text{solid}}$,