Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **17**, No. 3, pp. 1014-1036

DOI: 10.4208/aamm.OA-2022-0314
June 2025

Troubled-Cell Indicators using K-Means Clustering for RKDG Methods on Triangular Meshes and h-Adaptive Rectangular Meshes

Haiyun Wang[1], Zhen Gao[2], Haijin Wang[1], Qiang Zhang[3] and Hongqiang Zhu[1,*]

[1] *School of Natural Sciences, Nanjing University of Posts and Telecommunications, Nanjing, Jiangsu 210023, China*
[2] *School of Mathematical Sciences, Ocean University of China, Qingdao, Shandong 266100, China*
[3] *Department of Mathematics, Nanjing University, Nanjing, Jiangsu 210093, China*

Received 30 November 2022; Accepted (in revised version) 15 May 2023

Abstract. We have proposed a novel framework of troubled-cell indicator (TCI) using K-means clustering on uniform meshes in [SIAM J. Sci. Comput., 43 (2021), pp. A3009–A3031]. Based on this framework, we develop TCIs on two typical types of non-uniform meshes, i.e., triangular meshes and h-adaptive rectangular meshes. The TCIs are composed of two parts: one is to create the stencils for troubled-cell indication that are composed of computational cells in a local region, and the other is to detect the troubled cells stencil by stencil. Compared with the uniform meshes, the creation of stencils for non-uniform meshes is no longer trivial. We develop new stencil creation approaches specifically tailored to triangular meshes and h-adaptive rectangular meshes, respectively. Another contribution of this work is a new classification criterion in the troubled-cell indication part which is used to determine if there exist troubled cells in a stencil. It contains only one parameter, which leads to a much easier implementation of the TCIs. Numerical results show that the TCIs not only can capture the shocks precisely and produce nonoscillatory solutions, but also work well with multiple indication variables and in a TCI-based h-adaptive scheme. These results demonstrate the accuracy and robustness of the TCIs on non-uniform meshes.

AMS subject classifications: 65M60, 35L60, 35L65, 35L67

Key words: Troubled-cell indicator, shock detection, triangular mesh, adaptive mesh, discontinuous Galerkin method.

*Corresponding author.
Emails: wanghaiyun@stu.ouc.edu.cn (H. Wang), zhengao@ouc.edu.cn (Z. Gao), hjwang@njupt.edu.cn (H. Wang), qzh@nju.edu.cn (Q. Zhang), zhuhq@njupt.edu.cn (H. Zhu)

1 Introduction

The Runge-Kutta discontinuous Galerkin (RKDG) method [4–7] is one of the popular numerical methods for solving nonlinear hyperbolic conservation laws which can develop discontinuities in their solutions. As a finite element method, the DG method can handle complex geometries. In addition, it doesn't require continuity between elements, so it can handle arbitrary triangulation and hp-adaptation easily. The DG methods have found successful applications to many research fields [1, 29]. One important component of the RKDG method is to construct a limiter, which is used to control numerical oscillations for stability. The limiter is usually composed of two parts. The first part is a troubled-cell indicator (TCI), used to detect the cells in the vicinity of the discontinuities. The second part is a numerical reconstruction method to control spurious oscillations on detected cells. Accurate and robust TCIs have been actively pursued in the literature in the last decades. This paper also focuses on them.

There are various types of TCIs/limiters, such as the total variation bounded (TVB) limiter [4–7], the moment based limiters [2, 3], the monotonicity-preserving (MP) limiter [22], the modified MP limiter [21], the Harten's TCI [11] and KXRCF shock detector [13]. Qiu and Shu [17] compared these TCIs combined with a WENO solution reconstruction method [18]. Recently, several new TCIs have been proposed, such as the posteriori subcell-based TCI [8], the multiwavelet TCI [23], the outlier detection based TCI [24], the Fu-Shu TCI [10], the artificial neural network based TCIs [9, 19, 20, 27], etc. These TCIs show good performance in detecting high gradients so that spurious oscillations are well controlled, and some of them are free of problem-dependent parameters, making them easier to use.

One deficiency of most existing TCIs is that they work in a dimension-by-dimension fashion when applied to multi-dimensional problems and are tricky to be extended to non-uniform meshes. To overcome this problem, we designed a new TCI in [33] using K-means clustering [12], a simple and popular approach for partitioning a data set into $\mathcal{K}$ distinct, nonoverlapping clusters. This TCI first collects the troubled-cell indication values from a group of computational cells in a local region, called a stencil, and then applies a K-means clustering algorithm to the values to single out the troubled cells, utilizing the different magnitudes of the indication values between smooth and discontinuous regions. It can be split into two separate parts: the stencil creation part that groups the computational cells into stencils according to some rules, and the troubled-cell indication part that detects the troubled cells stencil by stencil. When applied to a 2D problem, this TCI is a truly 2D approach. The numerical results in [33] and [26] show that it can detect the high gradients accurately on uniform meshes.

The major contribution of this work is the development of TCIs on non-uniform meshes based on the TCI framework using K-means clustering [33]. We consider two typical types of non-uniform meshes as examples. The other types either can be handled similarly or will be considered in future work. The first type is the conforming triangular mesh (referred to as triangular mesh hereafter) which is a widely-used unstructured