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A Signal Separation Method Based on Instantaneous Frequency Embedded Continuous Wavelet Transform and Short-Time Fourier Transform

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Dedicated to the memory of Prof. Donggao Deng on the occasion of his 90th birthday

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Abstract. Modeling a non-stationary, multicomponent signal as a superposition of frequency components, each with a well-defined instantaneous frequency (IF), is crucial for extracting information, such as the underlying dynamics hidden within the signal. The synchrosqueezing transform (SST) has emerged as an alternative to empirical mode decomposition (EMD) for separating non-stationary signals. However, because the SST estimates the IFs of all frequency components based on a single phase transformation, its accuracy can be limited. To address this, SST variants based on the IF-embedded short-time Fourier transform (IFE-STFT) and the IF-embedded continuous wavelet transform (IFE-CWT) were developed.

More recently, a direct time-frequency method called the signal separation operation (SSO) was introduced for multicomponent signal separation. SSO bypasses the second step of the two-step SST method for component recovery and is based on variants of the STFT or CWT. In this paper, we propose a direct signal separation method by combining the SSO method with IFE-CWT and IFE-STFT, creating the IFE-CWT-based SSO (IWSSO) and the IFE-STFT-based SSO (IFSSO). Both IWSSO and IFSSO directly separate multicomponent signals without the squeezing operation inherent in SST. Our algorithms and techniques yield more accurate instantaneous frequency estimates and signal separation than conventional SSO or SST methods.

Key Words: Instantaneous frequency (IF) estimation, mode retrieval, signal separation operator based on IF–embedded continuous wavelet transform.

AMS Subject Classifications: 94A12, 42C40, 65160

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1 Introduction

Recently the synchrosqueezing transform (SST) has been developed to sharpen the time-frequency representation of a non-stationary signal and to recover $x_k(t)$ of a multicomponent signal in the form of (see [1,2] and refer to the references in [3] on the variants of SST)

$$x(t) = A_0(t) + \sum_{k=1}^{K} x_k(t), \quad x_k(t) = A_k(t)\cos\big(2\pi\phi_k(t)\big), \tag{1.1}$$

where $A_k(t) > 0$, $\phi_k'(t) > 0$ with $A_k(t)$ changing slowly.

To recover individual component $x_k(t)$, the SST method consists of two steps. First IF $\phi'{}_k(t)$ of $x_k(t)$ is estimated from the SST plane. Secondly, after IF was recovered, $x_k(t)$ is computed by a definite integral along each estimated IF curve on the SST plane. The reconstruction accuracy for $x_k(t)$ depends heavily on the accuracy of the IFs estimation carried out in the first step and the sharpness of SST. On the other hand, a direct time-frequency approach, called signal separation operator or signal separation operation (SSO) scheme, was introduced in [4] for multicomponent signal separation. SSO avoids the second step of the two-step SST method in signal separation. The linear chirp-based SSO model was proposed in [5] and theoretically analyzed in [6]. The SSO approaches in [4–6] are based on STFT or its variants. The SSO signal separation method based on the continuous wavelets has been studied in [7]. In addition, the SSO scheme was extended from the 2-dimensional time-frequency (or time-scale) space to the 3-dimensional space of the time-frequency-chirp rate (or the time-csale-chirp rate respectively) using the chirplet transform for the recovery of components with cross-over IFs [8–10].

In this paper, we propose a direct signal separation method by combining SSO with IFE-CWT and IFE-STFT together respectively to form the IFE-CWT-based SSO (IWSSO) and IFE-STFT-based SSO (IFSSO). Both IWSSO and IFSSO separate multicomponent signals directly, without squeezing operation being involved. Our algorithms result in more accurate component recovery than the coventional SSO or SST method.

The rest of the paper is organized as follows. We propose IWSSO and IFSSO in Sections 2 and 3 respectively. Experimental results are provided in Section 4.

2 Signal separation based on instantaneous frequency-embedded CWT

In this section, we first recall the defintion and some properties of the instantaneous frequency-embedded CWT (IFE-CWT). After that we propose a signal separation method by combining IFE-CWT and SSO together.