An Explicit Multi-Time Stepping Algorithm for Multi-Time Scale Coupling Problems in SPH

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Abstract. Simulating physical problems with multi-time scale coupling presents a considerable challenge due to the concurrent solution of processes with different time scales. This complexity arises from the necessity to evolve large time scale processes over long physical time, while simultaneously small time step sizes are required to unveil the underlying physics in shorter time scale processes. To address this inherent conflict in the multi-time scale coupling problems, we propose an explicit multi-time step algorithm within the framework of smoothed particle hydrodynamics (SPH), coupled with a solid dynamic relaxation scheme, to quickly achieve equilibrium state in the comparatively fast solid response process. To assess the accuracy and efficiency of the proposed algorithm, a manuscript torsional example, two distinct scenarios, i.e., a nonlinear hardening bar stretching and a fluid diffusion coupled with Nafion membrane flexure, are simulated. The obtained results exhibit good agreement with analytical solution, outcomes from other numerical methods and experimental data. With this explicitly multi-time step algorithm, the simulation time is reduced firstly by independently addressing different processes being solved under distinct time step sizes, which stands in contrast to the implicit counterpart, and secondly decreasing the simulation time required to achieve a steady state for the solid by incorporating the dynamic relaxation scheme.

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1 Introduction

Smoothed Particle Hydrodynamics (SPH), a typically mesh-free method, which is originally introduced by Lucy [1], Gigold and Monaghan [2] for studying astrophysical problems, has been widely applied to simulate fluid-flows [3–6], solid mechanics [7–11], fluid-structure interaction (FSI) [12–15], etc., in recent years. Comprehensive reviews can be found in Refs. [16–20]. Despite its broad applicability, SPH exhibits limitations when addressing multi-scale coupling problems prevalent in diverse engineering domains, particularly in scenarios involving the rapid dynamic response of solids [21]. The large disparity in the time scales between fast and slow processes posts a continuing challenge to numerical simulations [22].

To solve multi-time scale problems, the implicit, explicit or two schemes coupling can be applied [23,24]. The implicit scheme offers the advantage of using a larger time step in the time integration process [23,25,26], enabling the monolithic scheme to simultaneously solve the equations for both fast and slow processes. For instance, Zhao [27] utilized an implicit Newmark scheme to simulate fluid flow through a porous elastic solid, where solid dynamics and fluid diffusion occur at different time scales. Similarly, Gaston [28] employed an implicit scheme to analyze the coupling behavior among fluid dynamics, chemistry, and structural mechanics in a reactor. However, since the implicit scheme necessitates the inversion of the stiffness matrix for solving equations at each time step [29, 30], this approach incurs substantial computational costs and demands significant memory resources [31].

For enhancing computational efficiency, techniques involving explicit scheme are more favorable for addressing multi-time scale coupling problems due to its direct time integration and straightforward numerical formulation [32–35]. By partitioning the mesh into subdomains and the governing equations into subsystems, explicit-implicit and explicit-explicit partitions have been used to solve coupled-field dynamic problems, allowing multi-time step integrating with staggered solution procedures [36–38]. However, in systems displaying widely different characteristic response times, this methodology demands a large amount of staggered steps and data transfers per time step, rendering the treatment of realistic three-dimensional problems economically unfeasible [39]. Some researchers have employed explicit scheme to simulate material stretching and necking, where the load is applied over a long time period while the material's dynamic response is instant and fast [40–42]. Since the realistic load is applied in a long time scale, a correspondingly long physical simulation time is expected. However, with a quite small stable time step size allowed in explicit scheme for the fast process, a substantial number of time steps, often reaching millions, is typically necessary to simulate the entire process, presenting practical challenges. To reduce the overall simulation time, loading rate is usually increased artificially [41]. Nevertheless, the adoption of high non-realistic loading rate may lead to certain limitations and inaccuracies in the simulation results [43], underscoring the delicate balance required in managing loading rates for accurate simulations.