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A Data-Driven Reconstruction for Unstructured Finite Volume Method on Multi-Dimensional Compressible Flows

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Abstract. This paper introduces a novel data-driven shock-capturing for unstructured finite volume method (FVM) on multi-dimensional compressible flows, aiming at enhancing accuracy in smooth region while ensuring robustness in discontinuous region, by machine-learning models. We achieve this objective via a two-step process based on a tree model and fully-connected neural network (FCNN) models. In the first step, a tree model is employed as a troubled-cell indicator to divide the computational domain into discontinuous and smooth regions. In the second step, we employ FCNN models to reconstruct solution variables at a cell interface, which are used to obtain numerical fluxes. Notably, two FCNN models are used for discontinuous and smooth regions, respectively, with the former designed for robust capturing of discontinuities, and the latter intended to enhance accuracy in smooth region. To train such models, we generate two types of datasets using various analytic functions to mimic discontinuous and smooth FVM solutions. In addition, we define specific input features to provide the information on solution distribution and coordinates, enabling the extension onto unstructured meshes. Finally, we systematically validate the proposed method through a comprehensive set of numerical experiments, ranging from scalar conservation laws, Euler equations to Navier-Stokes equations. Computational experiments demonstrate enhanced performance in terms of robustness and accuracy, highlighting the capability of data-driven reconstruction in complex flow simulations.

AMS subject classifications: 36L65, 35L70, 68T07, 76M12

Key words: Finite volume method (FVM), shock-capturing method, data-driven reconstruction method, machine learning (ML), tree model, fully-connected neural network (FCNN).

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1 Introduction

In numerous engineering and scientific applications, finite volume method (FVM) that is essentially free from grid topology is widely adopted as a standard discretization for developing a flow solver. FVM can readily achieve high-order accuracy via high-order polynomial reconstruction, but oscillations leading to numerical instability can be ubiquitous in supersonic and hypersonic flows with strong shocks, due to violating the monotonicity constraint [1].

To overcome such barrier, many shock-capturing methods have been developed based on various stability conditions. Total Variation Diminishing (TVD) [2,3], and Total Variation Bounded (TVB) [4] conditions assert that the total variation of a solution does not increase or remains bounded over time. Representative TVB-based reconstruction methods include Essentially Non-Oscillatory (ENO) [5,6] and Weighted Essentially Non-Oscillatory (WENO) [7–11] methods. In multi-dimensional problems, extending TVD- and TVB-based schemes usually adopt the idea of dimensional splitting, compromising their ability to accurately reconstruct high-order polynomials on irregular meshes. The maximum principle has been introduced to ensure the solution positivity and boundedness on multi-dimensional unstructured meshes [12, 13]. Based on the local maximum principle, various shock-capturing methods have been developed [14–21]. Among them, the Multi-dimensional Limiting Process (MLP) limiter [20,21] realized the concept of dual limiting by exploiting the values of both the cell-center and cell-vertex on unstructured meshes, leading to the remarkable enhancement of robustness, accuracy, and convergence in multi-dimensional high-speed flows. For all such efforts, shock-capturing methods often lose accuracy across smooth extrema, and are not free from delicate control of key parameters.

Recently, much attention has been paid to data-driven approaches to improve the performance of shock-capturing methods from the view point of robustness and accuracy. Compared to traditional shock-capturing, data-driven methods offer complementary advantages, such as the exclusion of user-defined parameters, objective decision-making, or the capacity to train with the best possible combination of datasets.

Some earlier examples of data-driven shock-capturing methods include the design of a troubled-cell indicator [22, 23], which employs artificial neural networks (ANN) to detect shock discontinuities with a nodal discontinuous Galerkin method. Yu and Hesthaven [24] developed an ANN-based artificial viscosity model for high-order shock capturing. Beck et al. [25] applied edge detection techniques using convolutional neural network to classify cells in the shock region and to localize shock waves on the sub-cell level, achieving an accurate prediction of shock position and localization. Feng et al. [26,27] extended an ANN-based troubled-cell indicator trained with one-dimensional data to multi-dimensional shock wave flows via dimensional splitting. Discacciati et al. [28] introduced an artificial viscosity model that learns suitable artificial viscosity methods under various flow situations, eliminating the need for parameter tuning. In addition, Huang et al. [29, 30] employed an ANN for solving conservation laws with