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High-Order Cell-Centered Lagrangian Spectral Volume Method on Curvilinear Meshes

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Abstract. This paper introduces a novel high-order, cell-centered Lagrangian method tailored for curvilinear meshes. To ensure stable curvilinear mesh motions and avoid the algorithmic complexity associated with special mesh stabilization techniques, this method exploits the subcell-wise computational degrees of freedom inherent in the well-established spectral volume (SV) method. It is demonstrated that the numerical results obtained from the SV reconstruction can serve as direct inputs for nodal solvers, yielding favorable mesh node velocities. For cells potentially containing discontinuities, the classical finite volume (FV) reconstruction is applied to each subcell within these cells instead. This subcell-wise reconstruction, analogous to common cell-wise limitations or reconstructions, can effectively reduce unphysical oscillations. More importantly, it is capable of suppressing spurious mesh motions following the same principle of not introducing stabilization techniques such as input correction. The proposed Lagrangian method is completely designed in the computational space with transformed governing equations. Hence, both of the aforementioned SV and FV reconstructions can be performed with the same ease as on uniform and time-independent Eulerian meshes and achieve high-order accuracy. The method further benefits from simplicity and uniformity in its implementation, the compactness of the reconstruction formulae utilized, and the efficiency of the iteration-free nodal solver employed. It exhibits consistently satisfying performance across various numerical tests, including those involving significant mesh deformation and extreme conditions such as low-pressure states accompanied by strong shocks.

AMS subject classifications: 65M12, 76M12, 65M50

Key words: Lagrangian gas dynamics, high-order method, curvilinear mesh, spectral volume.

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1 Introduction

Based on different perspectives of describing fluid flows, the following categories of numerical methods have been developed by researchers and demonstrate their pros and cons: The Eulerian methods employ fixed meshes and compute the fluxes across mesh boundaries. High-quality meshes can be maintained during simulations, but they exhibit limitations in capturing flow structures and extending to multi-medium scenarios. The Lagrangian methods track fluid particles, thereby moving the meshes dynamically. They excel in resolving flow structures like interfaces and handling multi-medium problems. In elastoplastic simulations, they inherently retain material history variables. Nevertheless, mesh robustness, especially in flows with complex deformations, remains a significant challenge, and curvilinear meshes are required to achieve high-order accuracy [1, 2]. The arbitrary Lagrangian–Eulerian (ALE) methods [3–10] combine the features of the above two types of methods, allowing the meshes to possess relatively independent motions to improve quality. However, they are generally more complicated, as distinct phases (e.g., the Lagrangian phase, rezoning phase, and remapping phase) are unavoidably involved. The Eulerian-Lagrangian (EL) methods [11,12] are also appealing. They employ fixed meshes like the Eulerian approach and track flow characteristics in a Lagrangian-like manner (but the direction is backward in time) to update solutions. For greater flexibility, these characteristics are approximated, and curvilinear meshes can be incorporated. In terms of numerical stability or allowable time steps, they have demonstrated relaxed constraints [13].

The Lagrangian methods are focused on in the current study, which also provides a solid foundation for the Lagrangian phase in any ALE extension. The Lagrangian methods are commonly divided into two categories: the staggered ones and the cell-centered ones. The staggered methods [14–20] define velocity at mesh nodes and other variables (density, pressure, internal energy) at cell centers. Owing to the special definition of velocity, mesh motions can be uniquely determined. These methods have achieved great success in recent decades, but theoretical and practical deficiencies such as mesh imprinting and symmetry breaking still exist [21]. Some efforts in improving the mesh stability can be found in [22–24]. Besides, if a staggered method is adopted in the Lagrangian phase of an ALE algorithm, the subsequent remapping phase may encounter difficulties in devising appropriate remapping algorithms for both momentum and energy [25], since the variables are not conserved over the same space. On the contrary, the cell-centered Lagrangian methods define all physical variables over the same control volume around cell centers [26], making their ALE extension and conservation preservation much simpler. Furthermore, designing coherent high-order methods for all variables becomes possible. As a necessary consequence, mesh motions have to be determined either by least squares [27], averaging [1, 2], or nodal velocity solvers [25, 28–31] because physical velocities are over-determined at mesh nodes.

In the design of the above cell-centered Lagrangian methods, the finite volume (FV)