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Numerical Simulation of Power-Law Fluid Flow in a Trapezoidal Cavity using the Incompressible Finite-Difference Lattice Boltzmann Method

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Abstract. In this paper, a numerical investigation of power-law fluid flow in the trapezoidal cavity has been conducted by incompressible finite-difference lattice Boltzmann method (IFDLBM). By designing the equilibrium distribution function, the Navier-Stokes equations (NSEs) can be recovered exactly. Through the coordinate transformation method, the body-fitted grid in physical region is transformed into a uniform grid in computational region. The effect of Reynolds (Re) number, the power-law index n and the vertical angle θ on the trapezoidal cavity are investigated. According to the numerical results, we come to some conclusions. For low Re number $Re=100$, it can be found that the behavior of power-law fluid flow becomes more complicated with the increase of n. And as vertical angle θ decreases, the flow becomes smooth and the number of vortices decreases. For high Re numbers, the flow development becomes more complex, the number and strength of vortices increase. If the Reynolds number increases further, the power-law fluid will changes from steady flow to periodic flow and then to turbulent flow. For the steady flow, the lager the θ, the more complicated the vortices. And the critical Re number from steady to periodic state decreases with the decrease of power-law index n.

AMS subject classifications: 76M28, 76T06

Key words: Finite difference lattice Boltzmann method, coordinate transformation, power-law fluid, trapezoidal cavity.

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1 Introduction

Over the last couple of decades, tremendous amount of research has been carried out in solving NSEs, such as finite difference method [1], finite element method [2], finite volume method [3]. In particular, the lattice Boltzmann method (LBM), as a novel alternative method, has been widely concerned now [4–7]. As a classic benchmark problem described by NSEs, the two-dimensional lid-driven flow in a square cavity has also been widely investigated [8–11], including the study of high Reynolds number flow [12] and the three-dimensional lid-driven cavity flow [13]. On this basis, the lid-driven flows of different cavities shapes were also simulated. In 2006, Patil et al. [14] applied the lattice Boltzmann equation to simulate the lid-driven flow in a two-dimensional rectangular deep cavity. He studied several features of the flow, such as the location and strength of the primary vortex, and the corner-eddy dynamics. Then, Cheng et al. [15] investigated the vortex structure in a lid-driven rectangular cavity at different depth-to-width ratios and Reynolds numbers by a lattice Boltzmann method. Zhang et al. [16] used the lattice BGK model to simulate lid-driven flow in a two-dimensional trapezoidal cavity. In addition, Li et al. [17] presented an accurate and efficient calculations of the flow inside a triangular cavity for high Reynolds numbers. And Erturk et al. [18] studied the numerical solutions of 2-D steady incompressible flow in a driven skewed cavity.

However, all above works only studied the Newtonian fluids. Non-newtonian fluids are widely observed in nature and industrial production, such as petroleum, food, geophysics, lubricants, chemistry, hydrogeology, to name but a few [19]. Unlike the Newtonian fluid, the relationship between shear stress and shear strain rate of non-Newtonian fluid is nonlinear. As a result, the non-Newtonian fluid will show shear thickening and shear variation characteristics. Due to the complicated constitutive equation of non-Newtonian fluid, it is a challenge to investigate the non-Newtonian fluid behavior by numerical methods. Recently, there are many efforts have been made to simulate non-Newtonian fluid flows through LBM in various computational geometries [20–26], such as the Non-Newtonian flow through porous media [20], the filling of expanding cavities by Bingham fluids [21] and the non-Newtonian pseudo-plastic fluid in a micro-channel [25]. In addition, Gabbanelli et al. [22] studied the shear-thinning and shear-thickening fluids in parallel and reentrant geometries by LBM. Boy et al. [23] presented a second-order accurate LBM for the simulations of the power-law fluid in a two-dimensional rigid pipe flow. Yoshino et al. [24] developed a LBM to investigate the power-law model in a reentrant corner geometry and flows inside a three-dimensional porous structure. Mendu and Das [27] applied the LBM to study the power-law fluids inside a two-dimensional enclosure driven by the motion of the two facing lids. Psihogios et al. [28] investigated the non-Newtonian shear-thinning fluid flow in three dimensional digitally reconstructed porous domain. Hamedi and Rahimian [25] simulated the power-law model for pseudo-plastic fluids in micro-channel by using LBM. Wang and Ho [26] investigated the shear thinning non-Newtonian blood flows through LBM. Chai