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A Unifying Moving Mesh Method for Curves, Surfaces, and Domains Based on Mesh Equidistribution and Alignment

Min Zhang[1,2,*] and Weizhang Huang[3]

[1] *National Engineering Laboratory for Big Data Analysis and Applications, Peking University, Beijing, 100871, China.*
[2] *Chongqing Research Institute of Big Data, Peking University, Chongqing, 401121, China.*
[3] *Department of Mathematics, University of Kansas, Lawrence, Kansas 66045, USA.*

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Abstract. A unifying moving mesh method is developed for general m-dimensional geometric objects in d-dimensions ($d \geq 1$ and $1 \leq m \leq d$) including curves, surfaces, and domains. The method is based on mesh equidistribution and alignment and does not require the availability of an analytical parametric representation of the underlying geometric object. Mathematical characterizations of shape and size of m-simplexes and properties of corresponding edge matrices and affine mappings are derived. The equidistribution and alignment conditions are presented in a unifying form for m-simplicial meshes. The equation for mesh movement is defined based on the moving mesh PDE approach, and suitable projection of the nodal mesh velocities is employed to ensure the mesh points are not moved out of the underlying geometric object. The analytical expression for the mesh velocities is obtained in a compact matrix form. The nonsingularity of moving meshes is proved. Numerical results for curves ($m=1$) and surfaces ($m=2$) in two and three dimensions are presented to demonstrate the ability of the developed method to move mesh points without causing singularity and control their concentration.

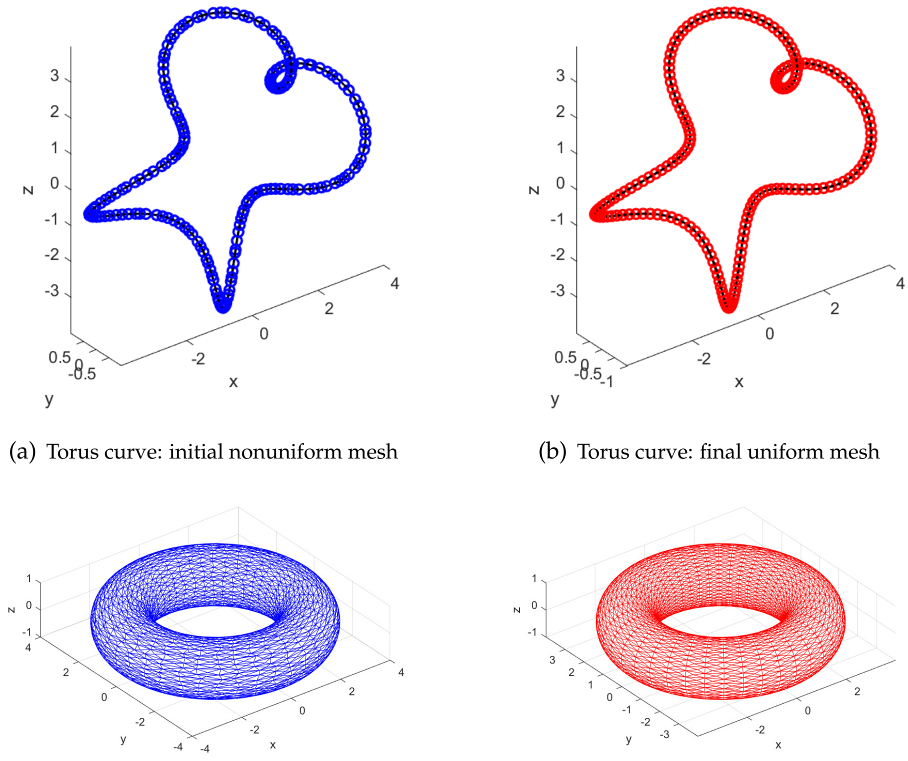
AMS subject classifications: 65M50, 65N50

Key words: Unifying method for mesh movement, moving mesh PDE, mesh nonsingularity, equidistribution, alignment.

1 Introduction

We are interested in the development of a unifying moving mesh method that can be used to move simplicial meshes on a general bounded m-dimensional geometric object S

*Corresponding author. *Email addresses:* `minzhang@pku.edu.cn` (M. Zhang), `whuang@ku.edu`(W. Huang)



(a) Torus curve: initial nonuniform mesh

(b) Torus curve: final uniform mesh

(c) Torus surface: initial nonuniform mesh

(d) Torus surface: final uniform mesh

Figure 1: Mesh examples for a torus curve and a torus surface in $\mathbb{R}^3$.

in $\mathbb{R}^d$ ($d \geq 1$ and $1 \leq m \leq d$) with or without an analytical parametric representation. Such a method is useful for the computation of the evolution of S and the numerical solution of partial differential equations (PDEs) defined on S. Notice that S can be a domain ($m = d$), a curve ($m = 1 < d$), or a surface ($m = 2 < d$) in $\mathbb{R}^d$. Moreover, the method does not require the availability of an analytical parametric representation of S. Generally speaking, it needs to use the information of the normal/tangent vector to ensure the mesh points are not moved out of S. The curvature of S (for $m < d$) is also needed if we want to control mesh concentration based on the curvature. These information can be obtained from a mesh that presents S reasonably accurately. Mesh examples are shown in Fig. 1 for a torus curve and a torus surface in $\mathbb{R}^3$. The initial meshes are nonuniform, generated with random perturbations to the location of mesh points. The final meshes are obtained by applying the unifying moving mesh method to be presented in this work with the geometric information of normal/tangent to S computed from the initial meshes and with a mesh concentration control in an attempt to make the mesh more uniform.