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A New Approximate Method to Mean Field Stochastic Differential Equation with One-Sided Lipschitz Drift Coefficient

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Abstract. In this paper, we construct a numerical method to approximate a class of mean field stochastic differential equations whose drift coefficient satisfies a one-sided Lipschitz condition. The key idea is to independently approximate the distribution via solving a nonlinear Fokker-Planck equation which governs the density function of the solution. Meanwhile, based on the approximate density function, we construct an approximate stochastic differential equation to approach the mean field one. Then, we also apply a truncated Euler-Maruyama method to achieve discretization and derive error estimates. Finally, we present several numerical experiments to illustrate our theoretical analysis.

AMS subject classifications: 65C30, 82C31, 60H35, 65N06

Key words: Mean field stochastic differential equation, nonlinear Fokker-Planck equation, truncated Euler-Maruyama method, error estimate.

1 Introduction

Mean field stochastic differential equation (SDE for short), whose coefficients depend on the solution and its distribution, is a powerful tool for modeling in various fields, cf. deep learning neural networks [14], molecular and fluid dynamics [26], mathematical biology [17] and plasma physics [4]. Moreover, in practical applications, the corresponding coefficient functions exhibit highly nonlinear behavior, such as the stochastic volatility model [21], the Hodgkin-Huxley and FitzHugh-Nagumo models [1, 5], and the generalized Ait-Sahalia model [30].

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From macroscopic point of view, the mean field SDE is often emanated from the limit of the average behavior of an interacting particle system, which characterizes the evolution of particles in the sense of statistical averages. Due to the lack of distribution of the unknowns, the numerical approximation to a mean field SDE is usually approached by a system consisting of a great number of interacting particles that are governed by a high dimensional SDE, where the distribution is replaced by an empirical measure [6,8,11,27]. The main idea of the particle method is to apply an empirical measure simulated by microscopic particles to approach the limit distribution and to employ a high-dimensional SDE to approximate the mean field SDE. However, as the number of particles increases, numerical calculation becomes difficult due to the high nonlinearity of the coefficients and the high cost of simulating large numbers of independent Brownian motions. Moreover, the computational cost of calculating the empirical measure becomes extremely high when the number of particles is large. In recent years, research on the mean field SDE has attracted extensive attention. Hammersley et al. investigated the weak solutions to the mean field SDE with common noise [13]. Wang et al. studied the exponential ergodicity for the mean field SDE [15,32,33]. Shao et al. proved the strong well-posedness for the conditional mean field SDE [28]. Sun et al. proposed an explicit second order scheme for decoupled mean field forward backward SDEs with jumps [29]. Zhou et al. [34] constructed a numerical method without using an empirical measure to approach the mean field SDE whose coefficients have bounded derivatives.

In many applications [2,16,23], the Lipschitz continuity of coefficients is unable to be achieved, therefore, we are concerned with mean field SDEs under non-Lipschitz conditions. More precisely, the objective of this work is to develop a new numerical method to approximate a d-dimensional mean field SDE with one-sided Lipschitz drift coefficient. As is well known, the density function of the solution to the mean field equation satisfies a nonlinear Fokker-Planck equation. One of our contributions is to calculate an approximate density function by applying an explicit-implicit difference scheme to the nonlinear Fokker-Planck equation and derive error estimates. Furthermore, we use the approximate density function to replace the unknown density function in mean field SDE and obtain a d-dimensional approximate SDE with nonlinear coefficients. The other contribution is to use the approximate SDE to explore the statistical properties of the mean field SDE, which largely reduces the computation complexity compared to the particle method in low-dimensional problems. In order to carry out simulations, we apply a truncated Euler-Maruyama method to the approximate SDE and derive error estimates.

This paper is organized as follows. In Section 2, we introduce the mean field SDE and some assumptions. In Section 3, we construct an effective numerical method to approximate the nonlinear Fokker-Planck equation and provide error estimates. In Section 4, we construct a new numerical method to approximate the mean field SDE and derive the corresponding error estimates. In Section 5, we provide some numerical experiments to illustrate the effectiveness of our method.