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Conservative Finite-Difference Scheme for High-Frequency Acoustic Waves Propagating at an Interface Between Two Media

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Abstract. This paper is an introduction to a conservative, positive numerical scheme which takes into account the phenomena of reflection and transmission of high frequency acoustic waves at a straight interface between two homogeneous media. Explicit forms of the interpolation coefficients for reflected and transmitted wave vectors on a two-dimensional uniform grid are derived. The propagation model is a Liouville transport equation solved in phase space.

AMS subject classifications: 35L45, 65M06, 65M12, 70H99

Key words: Acoustics, high-frequency, Liouville equation, energy conservation, reflection, transmission.

1 Introduction

The present work deals with propagation of high-frequency acoustic waves at a straight interface between two homogeneous media characterized by their respective celerities and densities. The main issue that is considered here is the conservation of the total energy by the finite difference scheme used for numerical simulations. The overall propagation problem is splitted into two sub-problems, first propagation of high-frequency waves in both media itself, and second their behaviour at the interface considering reflection and transmission. Propagation is described by a Liouville transport equation which rules the evolution of the acoustic energy density in time and phase space (position $\times$ wave vector) [12]. It is solved numerically by a finite difference scheme [2, 9] applied in time and phase space following the original developments of Jin and co-workers [3–7]. The behaviour of the waves at the interface is described by Snell-Descartes laws [1, 8].

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However the discontinuity of the celerities at that interface is a source of numerical dissipation, that is to say a net loss for the computed total acoustic energy. A refined interpolation of the acoustic energy density in the cells that have a common edge with the interface is necessary to obtain a conservative scheme. This interpolation has to be done on a wave-vector mesh which does not necessarily includes the reflected and/or transmitted wave vectors as given by Snell-Descartes laws. The main contribution of this paper is the derivation of an adapted finite difference scheme to circumvent this shortcoming. It is constructed in such a way that the (computed) overall acoustic energy remains conserved. The whole derivation is carried out for a two-dimensional acoustic medium, which is the plane of incidence of an acoustic source with a given incident wave vector. The diffraction phenomenon at critical incidence as considered by Jin and Yin [7] is however ignored in this work. It is the subject of ongoing research.

The physical model is recalled in Section 2, as well as the time and phase-space finite difference scheme introduced in [3–7]. The construction of a conservative scheme including reflection and transmission at the interface is outlined in Section 3. There the computation of the increment of the total acoustic energy between two successive time steps is performed and used as a guideline to construct explicitly an adapted interpolating expansion on the wave-vector grid. Both cases of transmission from the fast to the slow medium (Section 3.3), and from the slow to the fast medium (Section 3.4) are considered. Some comparisons of the proposed new scheme with the one of Jin and coworkers [3–7] are done through the numerical results given in Section 4. Finally Section 5 offers a few conclusions.

2 High-frequency acoustic wave propagation with a sharp interface

The transport model for high-frequency wave propagation in heterogeneous acoustic media derived in [12] is summarized in Section 2.1 below. It has been shown that the acoustic energy density associated to these waves satisfies a Liouville transport equation up to the interface. There it is reflected and/or transmitted according to Snell-Descartes laws [1] as recalled in Section 2.2. The last Subsection 2.3 describes how the Liouville equation is discretized by time and phase-space finite differences following an upwind scheme up to the interface.

2.1 Acoustic energy propagation

We consider the propagation of the high-frequency energy density in an acoustic medium $\mathcal{O} \subset \mathbb{R}^2$ divided into two subdomains $\mathcal{O}^-$ and $\mathcal{O}^+$ by a straight interface Γ oriented by its unit normal $\hat{\mathbf{n}}$. For convenience and without loss of generality this interface is the line $\Gamma = \{\mathbf{x} = (0,y),\ y \in \mathbb{R}\}$, thus $\mathcal{O}^- = \{\mathbf{x} = (x,y) \in \mathcal{O},\ x < 0\}$, $\mathcal{O}^+ = \{\mathbf{x} = (x,y) \in \mathcal{O},\ x > 0\}$ and we choose $\hat{\mathbf{n}} = (1,0)$. The tangent unit vector to the interface is denoted by $\hat{\mathbf{t}}$. The