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Two Uniform Tailored Finite Point Schemes for the Two Dimensional Discrete Ordinates Transport Equations with Boundary and Interface Layers

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Abstract. This paper presents two uniformly convergent numerical schemes for the two dimensional steady state discrete ordinates transport equation in the diffusive regime, which is valid up to the boundary and interface layers. A five-point node-centered and a four-point cell-centered tailored finite point schemes (TFPS) are introduced. The schemes first approximate the scattering coefficients and sources by piece-wise constant functions and then use special solutions to the constant coefficient equation as local basis functions to formulate a discrete linear system. Numerically, both methods can not only capture the diffusion limit, but also exhibit uniform convergence in the diffusive regime, even with boundary layers. Numerical results show that the five-point scheme has first-order accuracy and the four-point scheme has second-order accuracy, uniformly with respect to the mean free path. Therefore a relatively coarse grid can be used to capture the two dimensional boundary and interface layers.

AMS subject classifications: 65L12, 76N20, 35Q70

Key words: Neutron transport equation, discrete ordinates method, tailored finite point method, boundary layers, interface layers.

1 Introduction

The neutron or radiative transport equation is widely used in nuclear engineering, thermal radiation transport, charged-particle transport and oil-well logging tool design, etc..

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Developing efficient numerical methods for the neutron transport equation has been an active area for decades [22–24].

The solutions of the neutron transport equation depend on space, time, and velocities, which require a lot of computational cost for simulations. The discrete ordinates version of the steady state neutron transport equation is a semi-discretization in velocity. Starting from the discrete ordinates methods, which are among the most popularly used methods in the community, various space discretizations are investigated in the last two decades. For example, the diamond-difference method [24], the characteristic method [3, 7], the discontinuous finite element method [1, 29], the nodal method [2, 23], and so on.

When the average distance between two successive collisions (the mean free path) ϵ is small, it is generally impossible to accurately solve the discrete ordinates transport equation in the diffusive regime by optically thin ($\Delta x \ll \epsilon$) meshes, because of limits in computer memory. To approximate the solutions, some macroscopic models have been derived by asymptotic analysis [21], for example, the optically thin limit, the optically thick absorptive limit and the optically thick diffusive limit [22]. Here in this paper, we focus ourselves on the diffusive regime. Two criteria for designing accurate space discretizations for the discrete ordinates transport equation are 1) the order of their truncation error which guarantees the convergence and accuracy in the optically thin regime; 2) the discretization should converge to a discretization of the diffusion limit equation as the mean free path tends to zero [19, 20]. This gives the accuracy with optically thick cell ($\Delta x \gg \epsilon$) of a transport spatial discretization.

The idea of using unresolved cells to capture the macroscopic limit model has been successfully extended to more general applications, which is called asymptotic preserving schemes [8]. However the asymptotic preserving property only guarantees the accuracy of the diffusive region away from the boundary layer. One important issue is the scheme behavior in the presence of unresolved boundary/interface layers. In many applications, if a diffusive region is adjacent to a transport region, boundary and interface layers may appear. Flux changes rapidly across the boundary/interface layers, which requires sufficiently fine grids to capture these changes. It is usually impractical to prescribe a spatial grid that adequately resolves all boundary/interface layers. Therefore, it is desirable to design numerical schemes that are accurate across the boundary/interface layers, even if the spatial grids are not fine enough to resolve the fast variations.

The known schemes for the neutron transport equation that can capture the boundary layers with coarse meshes (meshes that do no resolve the fast variation) are restricted to the one dimensional case. For example, the spectral nodal method proposed in [5, 9], the domain decomposition method in [10] and the micro-macro decomposition method discussed in [25]. These methods are shown to be valid up to the boundary even if the boundary layers exist, but *only in one dimension*. Though higher dimensional extensions have been investigated in [2,5,28], the additional approximations for the transverse leakage terms make these higher dimensional extensions no longer able to accurately capture the fast changes in the boundary layers.

The difference between one and high dimensional boundary layer analysis is that,