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Inverse Obstacle Scattering in an Unbounded Structure

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Abstract. This paper is concerned with the acoustic scattering of a point incident wave by a sound hard obstacle embedded in a two-layered lossy background medium which is separated by an infinite rough surface. Given the point incident wave, the direct scattering problem is to determine the acoustic wave field for the given obstacle and infinite rough surface; the inverse scattering problem is to determine both the obstacle and the infinite rough surface from the reflected and transmitted wave fields measured on two plane surfaces enclosing the structure. For the direct scattering problem, the well-posedness is studied by using the method of boundary integral equations. For the inverse scattering problem, we prove that the obstacle and the infinite rough surface can be uniquely determined by the measured wave fields corresponding to a single point incident wave. To prove the local stability, the domain derivative of the wave field with respect to the change of the shapes of the obstacle and the infinite rough surface is examined. The local stability indicates that the Hausdorff distance of two domains is bounded above by the distance of corresponding wave fields if the two domains are close enough.

AMS subject classifications: 78A46, 78M30

Key words: Helmholtz's equation, inverse scattering problem, unbounded rough surface, domain derivative, uniqueness, local stability.

1 Introduction

This paper is concerned with the scattering of a point incident wave by an obstacle embedded in a two-layered background medium which is separated by an infinite rough surface. An obstacle is an impenetrable object which has a compact support; an infinite

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rough surface is referred to as a nonlocal perturbation of an infinite plane surface such that the whole surface lies within a finite distance of the original plane. Given the point incident wave, the direct scattering problem is to determine the wave field for the known composite medium consisting of the obstacle and the infinite rough surface; the inverse scattering problem is to determine simultaneously the obstacle and the infinite rough surface by the measured wave fields. The scattering problems arise in diverse scientific areas, such as modeling the optical scattering from obstacles and interfaces of materials in nano-optics, the acoustic wave propagation over the ground and sea surfaces in remote sensing, and the radar object recognition above the sea surface or detection of underwater mines.

Specifically, we study the scattering of a time-harmonic acoustic wave, which is generated by a point source, by a sound hard obstacle and an infinite rough surface. The space above and below the infinite rough surface is filled with a homogeneous and isotropic energy absorbing material, respectively. The wave propagation is modeled by the two-dimensional Helmholtz equation. The scattering problems in such a composite medium are challenging due to the unbounded nature of the interface and the nonlinearity associated with the inverse problem. These scattering problems have received considerable attention and a large amount of work have been done. The recent developments can be found in [25, 26, 33] on the shape and impedance parameter reconstruction in the inverse obstacle scattering problems. The characterization of domain derivatives with respect to local boundary disturbance were studied by Hettlich [20], Hiptmair and Li [22], Kirsch [23], Li [28], Potthast [38]. However, few results are available for rigorous analysis of the obstacle scattering problem in unbounded structures. Our goal is to examine mathematically both the direct and the inverse scattering problems in such a setting.

It is worth mentioning that considerable results are available for the scattering problems in unbounded structures. For unbounded periodic surfaces, many mathematical analysis and numerical computation have been done for both the direct and inverse scattering problems, see Bao [10], Bao, Cui and Li [11], Nédélec and Starling [37], Bao et al. [5] and references cited therein. For the local rough surface scattering problems, Li [28] considered an inverse cavity problem for Maxwell's equations, and showed the global uniqueness and a local stability to reconstruct the cavity wall. Ammari et al. [1] studied the method of integral equations for the electromagnetic scattering from open cavities. An optimization method was introduced in [3] to recover a local rough surface. In [4], a continuation approach over the wave frequency was developed for reconstructing a local rough surface. We refer to Bao et al. [2], Bao and Sun [6], Bao and Lai [9], Li et al. [30], Kress and Tran [24], Zhang and Zhang [42] for various mathematical and numerical methods to solve the local rough surface scattering problems. For the general unbounded non-periodic rough surfaces, the usual Sommerfeld or Silver-Müller radiation condition is not valid any more. Some appropriate radiation condition needs to be given as a part of the boundary value problem. In [16, 17, 41], Chandler-Wilde and Zhang proposed an upward going radiation condition to replace the usual Sommerfeld