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Troubled-Cell Indicator based on Mean Value Property for Hybrid WENO schemes

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Abstract. In this paper, we introduce a new type of troubled-cell indicator to improve hybrid weighted essentially non-oscillatory (WENO) schemes for solving the hyperbolic conservation laws. The hybrid WENO schemes selectively adopt the high-order linear upwind scheme or the WENO scheme to avoid the local characteristic decompositions and calculations of the nonlinear weights in smooth regions. Therefore, they can reduce computational cost while maintaining non-oscillatory properties in non-smooth regions. Reliable troubled-cell indicators are essential for efficient hybrid WENO methods. Most of troubled-cell indicators require proper parameters to detect discontinuities precisely, but it is very difficult to determine the parameters automatically. We develop a new troubled-cell indicator derived from the mean value theorem that does not require any variable parameters. Additionally, we investigate the characteristics of indicator variable; one of the conserved properties or the entropy is considered as indicator variable. Detailed numerical tests for 1D and 2D Euler equations are conducted to demonstrate the performance of the proposed indicator. The results with the proposed troubled-cell indicator are in good agreement with pure WENO schemes. Also the new indicator has advantages in the computational cost compared with the other indicators.

AMS subject classifications: 65M06, 65M12, 41A10, 35L65

Key words: WENO approximation, up-wind linear approximation, troubled-cell indicator, hyperbolic conservation laws, hybrid schemes.

1 Introduction

The hyperbolic equations are used to describe time-dependent phenomena and have been widely used in scientific researches and aeronautical applications such as in aero-

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dynamics, gas dynamics, astrophysical models and multiphase flow problems. It is well known that hyperbolic equations allow discontinuities even for smooth initial conditions in finite time. It is of great importance goals to develop high-order of accurate and efficient numerical methods for solving hyperbolic equations without spurious oscillations near discontinuities.

Many researchers have been developing numerical methods over the last decades to solve the hyperbolic system of equations; the total variation diminishing (TVD) scheme [9], the essentially non-oscillatory (ENO) scheme [10] and the weighted ENO (WENO) scheme [14] are very successful methodologies for solving conservation laws with strong discontinuities. The TVD scheme [9] uses limiters to suppress oscillations. The limiters reduce order of accuracy near discontinuities to efficiently eliminate oscillations; the sophisticated structures are often smeared since the accuracy degenerates to first order near smooth extrema.

Harten, Engquist, Osher and Chakravarthy introduced the cell average version of the ENO scheme [10] to get uniform high-order accuracy in smooth regions and non-oscillatory properties near discontinuities. The key idea of the ENO scheme is to use the smoothest stencil among all possible candidate stencils. Since the cell average version of the ENO scheme has disadvantages in multidimensional problems, Shu and Osher proposed the finite difference version of ENO scheme [26]. The improved version of the ENO scheme proposed by Liu, Osher and Chan [20], which is called WENO, uses a nonlinear convex combination of lower order polynomials obtained in each substencil to keep the accuracy in smooth regions and to suppress spurious oscillations near discontinuities. Later, Jiang and Shu [14] proposed the finite difference form of WENO (hereafter referred as WENO-JS) with improved smoothness indicator using the square sum of all derivatives to achieve $(2n-1)$th-order accuracy in smooth areas. The WENO method is widely used in the analysis of sophisticated flow structures with strong discontinuities because it is accurate in smooth areas and stable near discontinuities. However, WENO-JS is rather dissipative to solve problems having complicated structures such as the direct numerical simulation of turbulence. Henrick, Aslam, and Powers [12] introduced a simple nonlinear mapping for the weights of WENO-JS to yield the optimal convergence order at critical points in a smooth solution. This approach reduces the numerical dissipation and provides sharper results near discontinuities. In addition, several improved versions of the WENO method have been proposed to achieve higher order accuracy and less numerical dissipation for conservation laws [1,3,8]. Applying the WENO method to systems of conservation laws results in very high computational cost due to calculations of nonlinear weights from local characteristic decomposition/recomposition requiring matrix multiplications.

Jiang and Shu [14] tried to get nonlinear weights from entropy or pressure in order to save the computational cost of characteristic decomposition. Pirozzoli proposed hybrid compact-WENO scheme [21] based on the hybridization between fifth-order compact upwind scheme coupled with WENO scheme. Hill and Pullin developed a hybrid method which is tuned center-difference-WENO method [13]. Costa and Don proposed a high-