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An Improved Diffuse-Interface Lattice Boltzmann Method for Particulate Flows

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Abstract. In this paper, an improved diffuse-interface lattice Boltzmann method for particulate flows is developed. In this method, the difference in viscosity between the fluid and the particle is considered, and a smooth function is introduced to the collision term of the evolution equation. Some simulations are performed to test the method, and the numerical results show that, compared to the previous diffusion-interface lattice Boltzmann method [J. Liu et al., Comput. Fluids 233 (2022) 105240] which only has a first-order accuracy in space, the present one has a second-order convergence rate in space.

AMS subject classifications: 76M28, 76T25, 41A25

Key words: Diffuse-interface lattice Boltzmann method, particulate flows, convergence rate.

1 Introduction

Particulate flows are widely encountered in nature and industrial applications, such as watercourse silt, oil cracking, fluidized beds, and so on [1–3]. In the last three decades, the lattice Boltzmann method (LBM), as a kinetic-based numerical approach, has gained great success in the study of complex hydrodynamic problems [4–7]. Compared to the traditional numerical methods, the LBM has some distinct advantages, including the

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clear physical background, easy implementation of complex boundary conditions, natural parallelism in algorithm and simplicity in programming [6]. Due to these advantages of the LBM, it has also been applied to investigate particulate flows [8–11].

In the framework of LBM, there are mainly three kinds of numerical approaches that have been used to handle the fluid-particle interfaces, i.e., the bounce-back based method [9], immersed boundary-LBM (IB-LBM) [12, 13] and the diffuse-interface LBM (DI-LBM) [14–18]. Ladd [9] first proposed a shell method to simulate particulate flows. In this method, the inside of the particle is filled with fluid, the particle interface is represented by a set of midpoints between two fixed lattice nodes, and the boundary conditions of the fluid-particle interfaces are treated with the bounce-back scheme. Feng and Michaelides [12] developed an IB-LBM for fluid-particle interaction problems, which combined the LBM and the IB method proposed by Peskin [13]. In the IB-LBM, a set of Lagrangian points is introduced to denote the particle boundary, while the Euler mesh is adopted for the fluid domain. The fluid-particle interaction is realized by adding an external force to the fluid where the Dirac function is used.

In the DI-LBM [14–18], the fluid is filled with the whole domain, and the sharp boundary between the fluid and particle is replaced by a diffuse interface with a nonzero thickness, in which the physical variables are assumed to change smoothly. In addition, the fluid-particle interaction is realized through introducing a smooth function and modifying the force term in the evolution equation. Although there are some different kinds of DI-LBM, we only focus on the one proposed by Liu et al. [17, 18] since it is not only easy to implement for particulate flows, but also more suitable for the particulate flow with a complex geometry. It should be noted that, however, the DI-LBM [17] only has a first-order accuracy in space, which is mainly caused by the same viscosity used for both particle and fluid. To overcome this problem, in this work, we consider the difference in viscosity between the fluid and the particle, and develop an improved DI-LBM for particulate flows.

The rest of the paper is organized as follows. In Section 2, an improved DI-LBM for fluid-particle interaction problems is proposed. In Section 3, some numerical examples are carried out to test the improved DI-LBM, and finally, some conclusions are given in Section 4.

2 Numerical method

In this section, the LBM and the original DI-LBM are briefly introduced first, and then an improved DI-LBM is developed for particulate flows.

2.1 Lattice Boltzmann method

In the LBM for fluid flow problems, the evolution equation with the Bhatnagar-Gross-Krook (BGK) collision model can be expressed as [4]