Commun. Comput. Phys.
doi: 10.4208/cicp.030210.240910a

Vol. **10**, No. 2, pp. 433-452
August 2011

Modeling Magma Dynamics with a Mixed Fourier Collocation — Discontinuous Galerkin Method

Alan R. Schiemenz[1,2,*†], Marc A. Hesse[3] and Jan S. Hesthaven[2]

[1] *Department of Geological Sciences, Brown University, Providence, RI 02910, USA.*
[2] *Division of Applied Mathematics, Brown University, Providence, RI 02910, USA.*
[3] *Department of Geological Sciences, University of Texas at Austin, Austin, TX 78712, USA.*

Received 3 February 2010; Accepted (in revised version) 24 September 2010

Available online 28 April 2011

Abstract. A high-order discretization consisting of a tensor product of the Fourier collocation and discontinuous Galerkin methods is presented for numerical modeling of magma dynamics. The physical model is an advection-reaction type system consisting of two hyperbolic equations and one elliptic equation. The high-order solution basis allows for accurate and efficient representation of compaction-dissolution waves that are predicted from linear theory. The discontinuous Galerkin method provides a robust and efficient solution to the eigenvalue problem formed by linear stability analysis of the physical system. New insights into the processes of melt generation and segregation, such as melt channel bifurcation, are revealed from two-dimensional time-dependent simulations.

AMS subject classifications: 65Z05, 65M22, 65M70, 65T40, 65T50

Key words: Magma dynamics, discontinuous Galerkin method, Fourier collocation method, numerical simulations, linear stability analysis.

1 Introduction

Within the mantle of the Earth, melt is spontaneously generated and then segregated from its source, migrating upwards towards the Earth's surface. Melt is generated by adiabatic decompressional melting during upwelling in the Earth's upper mantle. Segregation of melt involves two-phase flow in which low viscosity melt percolates through a much more viscous solid matrix [3]. During its upward migration, melt generated in the

*Corresponding author. *Email addresses:* alan.schiemenz@geophysik.uni-muenchen.de (A. R. Schiemenz), mhesse@jsg.utexas.edu (M. A. Hesse), Jan.Hesthaven@brown.edu (J. S. Hesthaven)
†Current address: Department of Earth and Environmental Sciences, Geophysics, Munich University, Theresienstr. 41, 80333 Munich, Germany.

deeper part of the upwelling mantle will interact both thermally and chemically with the overlying mantle. Our understanding of the evolution of melt is constrained by physical evidence based upon field observations at ophiolites, where sections of the Earth's mantle have been uplifted and exposed at the surface. The key observation from these outcrops is the local depletion of a soluble mineral, orthopyroxene (opx). This observation has been interpreted as the localization of porous melt flow into high porosity melt channels that have lead to the depletion of the soluble mineral. The main objective of the present study is to develop a high-order numerical model that can be used to study the interaction between the melt and solid mantle during melt migration. Unlike previous studies we explicitly track the evolution of the soluble mineral (opx) to understand the formation of the observed opx-depletions and their spatial and temporal relation to localized melt flow in regions of high permeability.

A fundamental geologic observation that gives insight into the existing theory of melt transport is that the chemical composition of erupted basalt is not in equilibrium with residual mantle at low pressure, particularly at diverging plate boundaries. Additionally, it is observed that the mantle is chemically and lithologically heterogeneous. To preserve the geochemical signature developed at depth, melt must rise from depths of at least 30 km to the surface without extensive re-equilibration with the surrounding mantle. One mechanism for this involves reactive infiltration instability, where highly porous regions form due to a feedback between melt flow and dissolution of a soluble mineral in the solid. It has been suggested that high-permeability dunite channels act as conduits for focused flow, where melt may efficiently segregate from its source region while still maintaining its geochemical signature at depth. For further discussion we refer to [9] and the references therein.

Previous numerical study utilizing a low-order finite difference scheme was presented in [13], and demonstrated localization of the melt flow into high-porosity channels. However, this work does not explicitly account for the soluble mineral opx, whose presence is integral to the formation of dunite channels, nor does it consider the effects of upwelling or a bulk viscosity dependent upon the porosity. Further numerical study incorporating upwelling of the mantle was given in [12], although this work does not discuss the numerical model used. In this paper, a high-order accurate numerical model is presented following the physical models presented in [6,11], which include upwelling, a porosity-dependent bulk viscosity, and an additional equation to track the opx abundance. We assume a formulation of local chemical equilibrium with negligible diffusion in the melt. Linear analysis of our system predicts the emergence of compaction-porosity waves; in addition to the balancing between upwelling, dissolution and compaction, these features present a formidable challenge in numerical modeling. We address this challenge by developing a high-order numerical method that provides accurate resolution at a reasonable cost. Numerical simulations confirm the existence of compaction-porosity waves, and also demonstrate that melt channels are not necessarily spatially or temporally correlated with dunite channels.

This paper is organized as follows. In Section 2 we detail the non-dimensionalized