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Corrigendum to: "On the Eigenvalue Problem for a Bulk/Surface Elliptic System"

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Abstract. This note corrects a mistake in the paper "On the eigenvalue problem for a bulk/surface elliptic system" by the author, appeared in [Vitillaro, Commun. Math. Anal. Appl. 4 (2025)].

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Some of the main results in the paper [2] are stated under the assumption that Ω is a bounded open subset of $\mathbb{R}^N$ ($N \geq 2$) with C^1 boundary. The author recently realized that Ω must also be connected for these results to hold.

Indeed, Theorems 1.1, 1.2 and 3.1 in [2] may fail to hold when Ω is disconnected, depending on the location of the part Γ_0 of the boundary where the Dirichlet homogeneous condition is posed. For example, let us consider the case when $\Omega = \Omega_1 \cup \Omega_2$, with Ω_1 and Ω_2 open and $\overline{\Omega_1} \cap \overline{\Omega_2} = \emptyset$, and $\Gamma_0 \subseteq \partial\Omega_2$. In this case 0 is an eigenvalue for problem (1.1) in [2], having the characteristic function of Ω_1 as an eigenfunction. Hence Theorems 1.1 and 1.2 in this case are wrong. Also Theorem 3.1 is wrong, since uniqueness of solutions of (1.13) cannot be asserted.

Therefore, the assumption that "Ω is a bounded open subset of $\mathbb{R}^N$" must be replaced in paper [2] by "Ω is a bounded connected open subset of $\mathbb{R}^N$" (this assumption appears in two instances, in Abstract and at the beginning of Section 1.1). Under this new assumption, all the results in paper [2] hold.

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Let us now explain why this connectedness assumption is needed in paper [2]. The whole of the paper is based on the assertion, made in Section 2.5, that the inner product

$$(u,v)_{H^1} = \int_\Omega \nabla u \nabla \overline{v} + \int_{\Gamma_1} (\nabla_\Gamma u, \nabla_\Gamma v)_\Gamma, \quad \forall u,v \in H^1, \tag{0.1}$$

induces on the space

$$H^1 = \left\{ (u,v) \in H^1(\Omega) \times H^1(\Gamma) : v = u_{|\Gamma}, v = 0 \text{ on } \Gamma_0 \right\}$$

a norm $\|\cdot\|_{H^1} = (\cdot,\cdot)^{1/2}$, which is equivalent to the norm $|||\cdot|||_{H^1}$ given by

$$|||u|||_{H^1}^2 = \|\nabla u\|_2^2 + \|\nabla_\Gamma u\|_{2,\Gamma_1}^2 + \|u\|_{2,\Gamma_1}^2. \tag{0.2}$$

This assertion is correct only when Ω is connected, since the previous example shows that (0.1) does not even defines an inner product if Ω is not connected.

In [2] the assertion was justified by referring to [1, Lemma 1], where it is stated. The proof of [1, Lemma 1] was the following one. By [3, Chapter 2, Theorem 2.6.16, p. 75] one derives that, when $\mathcal{H}^{N-1}(\Gamma_0) > 0$, as in [2], the capacity $B_{1,2}(\Gamma_0)$ is positive. One then applies [3, Chapter 4, Corollary 4.5.2, p. 195] to conclude that the following Poincaré-type inequality holds: There is a positive constant $C_1 = C_1(\Omega,\Gamma_0)$ such that

$$\|u\|_2 \le C_1 \|\nabla u\|_2 \tag{0.3}$$

for all $u \in H^1(\Omega)$ such that $u_{|\Gamma} = 0$ on Γ_0. By the trace theorem there is a positive constant $C_2 = C_2(\Omega,\Gamma_0)$ such that

$$\|u\|_{2,\Gamma_1} \le C_2 \|u\|_{H^1(\Omega)}, \quad \forall u \in H^1(\Omega), \tag{0.4}$$

where $\|\cdot\|_{H^1(\Omega)}$ is the standard norm of $H^1(\Omega)$. Since $H^1 \subset H^1(\Omega)$, by combining (0.2)-(0.4) one obtains

$$|||u|||_{H^1}^2 \le \|\nabla u\|_2^2 + \|\nabla_\Gamma u\|_{2,\Gamma_1}^2 + C_2^2 (1 + C_1^2) \|\nabla u\|_2^2 \le C_3^2 \|u\|_{H^1}^2$$

for all $u \in H^1$, where $C_3^2 = 1 + C_2^2(1 + C_1^2)$, from which the statement trivially follows.

Unfortunately, the author has not noticed, before the publication of [2], that [3, Chapter 4, Corollary 4.5.2, p. 195] is obtained as a consequence of [3, Chapter 4, Theorem 4.5.1, p. 195], where Ω is assumed to be connected. Hence the connectedness assumption has to be added to the other assumptions on Ω made in the paper [2].