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# Classical Solutions to a Model for Heat Generation During Acoustic Wave Propagation in a Standard Linear Solid

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**Abstract.** In an open bounded real interval, this manuscript studies the evolution system

$$\begin{cases} u_{ttt}+\alpha u_{tt}=\big(\gamma(\Theta)u_{xt}\big)_x+\big(\widehat{\gamma}(\Theta)u_x\big)_x, \\ \Theta_t=D\Theta_{xx}+\Gamma(\Theta)u_{xt}^2, \end{cases}$$

which arises as a model for the generation of heat during propagation of acoustic waves in a standard linear solid.

A statement in local existence and uniqueness of classical solutions is derived for arbitrary $D>0$ and $\alpha\ge 0$, for sufficiently smooth $\gamma,\widehat{\gamma}$ and $\Gamma$ with $\gamma>0,\widehat{\gamma}>0$ and $\Gamma\ge 0$ on $[0,\infty)$, and for all suitably regular initial data of arbitrary size.

**AMS subject classifications**: Primary: 74H20; Secondary: 74F05, 35L05

**Key words**: Nonlinear acoustics, thermoelasticity, smooth solution, standard linear solid.

## 1 Introduction

This manuscript is concerned with the initial-boundary value problem

$$\begin{cases} u_{ttt}+\alpha u_{tt}=\big(\gamma(\Theta)u_{xt}\big)_x+\big(\widehat{\gamma}(\Theta)u_x\big)_x, & x\in\Omega, \quad t>0, \\ \Theta_t=D\Theta_{xx}+\Gamma(\Theta)u_{xt}^2, & x\in\Omega, \quad t>0, \\ u_x=0, \quad \Theta_x=0, & x\in\partial\Omega, \quad t>0, \\ (u,u_t,u_{tt},\Theta)(x,0)=(u_0,u_{0t},u_{0tt},\Theta_0)(x), & x\in\Omega, \end{cases} \tag{1.1}$$

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which arises as a simplified model for heat generation during acoustic wave propagation in so-called standard linear solids. Indeed, assuming a one-dimensional material to be of Zener type suggests to consider the displacement variable $u$ to be determined by the third-order hyperbolic equation in (1.1), where recent experimental observations suggest certain dependencies on the temperature $\Theta$ in the elastic parameters $\widehat{\gamma}$ and $\gamma$ related to stiffness and viscosity near relevant operating points [24]. In slight modification of a more complex model, (1.1) moreover assumes elastic behavior to be dominant as compared to mechanical losses, so that additional and temporally nonlocal contributions to the source in the heat subsystem of (1.1) can be neglected. For more details on the modeling background, we may refer to [9, 17]; we note that of predominant application relevance in purely thermomechanical settings seem scenarios in which $\Gamma \sim \gamma$, while including interaction with electric fields, such as of interest in the modeling of heat generation in piezoceramics, leads to choices with $\widehat{\gamma}(\Theta) = a\gamma(\Theta) + d$ with some $a > 0$ and $d > 0$ (see, e.g. [22, 44] for corresponding considerations detailed for a second-order relative of (1.1)).

In contrast to classical models for thermoviscoelastic evolution in Kelvin-Voigt materials which have undergone a thorough mathematical analysis over the past decades [4, 8, 18, 33, 39, 40], third-order models of the form in (1.1) seem to have become objects of study only in more recent literature. Despite the potential of the Zener modeling approach to appropriately capture key properties of standard linear solids such as instantaneous elastic responses and stress relaxation, a corresponding description by means of Moore-Gibson-Thompson equations such as in (1.1) to date seems to have mainly concentrated on linear cases (cf., e.g. [1, 3, 21, 42]).

The mathematical analysis of acoustic problems containing Moore-Gibson-Thompson type subsystems thus seems to have focused on models for wave propagation in non-solid materials in which $u$ can be chosen to represent a scalar pressure variable so far, and in which interaction with temperature fields, if of relevance at all, commonly leads to different types of couplings. Accordingly, analytical studies have been concerned with aspects of stability and large time decay in linear Moore-Gibson-Thompson problems in various special frameworks both in bounded $n$-dimensional domains [11, 20, 26, 31] and the entire space $\mathbb{R}^n$ [11, 12], partially also including certain memory terms [2, 19, 29, 30]; studies on nonlinear relatives have mainly addressed semilinear situations that involve various types of source terms depending either on the solution itself or some of its derivatives, focusing either on questions of local and global solvability [27, 28, 38, 41], or on the occurrence of blow-up phenomena [11, 13, 14, 34, 37].