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Reflected BSDEs Driven by RCLL Martingales with Stochastic Lipschitz Coefficient in a General Filtration: Analysis and Applications

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Abstract. In this paper, we investigate reflected backward stochastic differential equations with a single, discontinuous barrier, driven by a right-continuous, left-limited martingale within a general filtration. We establish the existence and uniqueness of solutions under a stochastic Lipschitz condition on the generator and a reflection process that is right-continuous with left limits. As an application, we use these results to determine fair pricing for American contingent claim options in a financial market driven by Azéma's martingale, incorporating elements of asymmetric information.

AMS subject classifications: 60H05, 60H10, 60H30

Key words: Reflected BSDEs, RCLL martingales, stochastic Lipschitz coefficient, Azéma's martingale market model, American option.

1 Introduction

The notion of one barrier reflected backward stochastic differential equations (RBSDEs) was introduced by El Karoui *et al.* [13] within the framework of a Brow-

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nian filtration associated with a standard Brownian motion $(B_t)_{t\leq T}$. In this framework, a solution to this equation, defined for a horizon time $0<T<\infty$, a coefficient f, a terminal value ξ, and a barrier $L=(L_t)_{t\leq T}$ is represented by a triple of adapted processes $(Y_t,Z_t,K_t)_{t\leq T}$ such that

$$Y_t=\xi+\int_t^T f(s,Y_s,Z_s)ds+(K_T-K_t)-\int_t^T Z_s dB_s, \quad Y_t\geq L_t, \quad \forall t\leq T. \tag{1.1}$$

In other words, the solution to the equation satisfies a certain differential equation and a reflecting boundary condition. In the Eq. (1.1), the non-decreasing continuous process K is introduced to keep the state process Y above the barrier L in a minimal energy. The process K only comes into play when Y reaches the obstacle L, as indicated by the condition $\int_0^T (Y_s-L_s)dK_s=0$. El Karoui *et al.* [13] have shown that when the terminal value ξ is square integrable, the coefficient f is uniformly Lipschitz with respect to (y,z), and the barrier L is continuous, the equation has a unique solution.

The literature on RBSDEs has seen numerous significant contributions. For instance, Hamadène and Lepeltier [26] study the case where the barrier L is right-continuous and left-upper semi-continuous, which implies positive jumps of L, and also explore the applications of RBSDEs in mixed control or zero-sum games. Meanwhile, Matoussi [36] establishes the existence of a solution to Eq. (1.1) when the driver f has linear growth and is only continuous with respect to (y,z). Building on these works, Hamadène [25] investigates discontinuous RBSDEs with a right-continuous and left-limited (RCLL) barrier under the same consideration made in [13], linking the findings with stochastic mixed control problems.

RBSDEs have been studied in various stochastic settings beyond the classical Brownian motion framework. Hamadène and Ouknine [27] considered the case where the filtration is generated by a Brownian motion and an independent Poisson point process. They allowed the obstacle to have only inaccessible jumps and established the existence and uniqueness of the reflected solution under the Lipschitz assumption on the coefficient. This work was later extended in [22, 28] to include barriers with general jumps that can be inaccessible or predictable. Other interesting results on RBSDEs can be found in [15, 34].

The objective of this paper is to explore the problem of existence and uniqueness for RBSDEs driven by a fairly RCLL general martingale with one reflected RCLL obstacle in an arbitrary filtered probability space. Our work builds upon and expands upon the results presented in previous studies such as [7, 12, 13, 15, 22, 25–28] (see also [19, 32, 42] for other related works). However, our approach is more comprehensive and we encounter several challenges in our problem, including: