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Global Solution of Euler-Poisson System in the Inviscid Limit of Navier-Stokes-Poisson System with General Density Dependent Viscosities

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Dedicated to Professor Gui-Qiang G. Chen on the occasion of his 60th birthday

Abstract. Based on delicate construction of approximate initial data sequence, elaborate estimates on the viscous terms and elegant analysis of the convergence of momentum equation, we succeed in establishing the global existence of spherically symmetric finite-energy weak solution of the compressible Euler-Poisson equations for large initial data through justifying the inviscid limit of the solutions of Navier-Stokes-Poisson equations with a very general class of density-dependent viscosities. Our results can serve as an extension of [Chen *et al.*, Comm. Pure Appl. Math. 77(6) (2024)].

AMS subject classifications: 35Q85, 85A30, 35L65, 35D30, 35Q31, 76N10

Key words: Euler-Poisson system, Navier-Stokes-Poisson system, inviscid limit, density dependent viscosities, spherical symmetry.

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1 Introduction and main result

The motions of the Newtonian self-gravitating gaseous stars are usually described by the three dimensional compressible Euler-Poisson equations (CEPEs), which reads

$$\begin{cases} \partial_t\rho+\nabla\cdot\mathcal{M}=0, \\ \partial_t\mathcal{M}+\nabla\cdot\left(\dfrac{\mathcal{M}\otimes\mathcal{M}}{\rho}\right)+\nabla P+\rho\nabla\Phi=0, \\ \Delta\Phi=\rho \end{cases} \tag{1.1}$$

for $(t,\mathbf{x}):=(t,x_1,x_2,x_3)\in\mathbb{R}_+\times\mathbb{R}^3:=(0,\infty)\times\mathbb{R}^3$. Here ρ is the density, $P=P(\rho)$ is the pressure, $\mathcal{M}\in\mathbb{R}^3$ is the momentum, Φ represents the gravitational potential of gaseous stars. For simplicity of presentation, we assume the constitute pressure-density relation to be the form of polytropic gaseous stars

$$P(\rho)=\kappa\rho^\gamma \quad \text{for} \quad \kappa>0, \quad \gamma\in\left(\frac{6}{5},3\right). \tag{1.2}$$

The restriction $\gamma>6/5$ is necessary to ensure the global existence of finite-energy solutions with finite total mass. Such a condition is also needed for the existence of the Lane-Emden solutions, see [4,27].

For the Cauchy problem of (1.1), we supplement (1.1) with following spherically symmetric initial data:

$$(\rho,\mathcal{M})(0,\mathbf{x})=(\rho_0,\mathcal{M}_0)(\mathbf{x})=\left(\rho_0(r),m_0(r)\frac{\mathbf{x}}{r}\right) \ \to \ (0,\mathbf{0}) \quad \text{as} \quad r:=|\mathbf{x}| \ \to \ \infty \tag{1.3}$$

subject to following far field conditions:

$$\Phi(t,\mathbf{x})=\Phi(r) \ \to \ 0 \quad \text{as} \quad r=|\mathbf{x}| \ \to \ \infty. \tag{1.4}$$

In the present paper, we are concerned with constructing the global existence of weak solution of the Cauchy problem of CEPEs (1.1), (1.3) and (1.4) via the vanishing viscosity limit of the solutions of following compressible Navier-Stokes-Poisson equation (CNSPEs) with density dependent viscosities:

$$\begin{cases} \partial_t\rho^\varepsilon+\nabla\cdot\mathcal{M}^\varepsilon=0, & (1.5\text{a}) \\ \partial_t\mathcal{M}^\varepsilon+\nabla\cdot\left(\dfrac{\mathcal{M}^\varepsilon\otimes\mathcal{M}^\varepsilon}{\rho}\right)+\nabla P^\varepsilon+\rho\nabla\Phi^\varepsilon & \\ =\varepsilon\nabla\cdot\left(\mu(\rho^\varepsilon)D\left(\dfrac{\mathcal{M}^\varepsilon}{\rho^\varepsilon}\right)\right)+\varepsilon\nabla\left(\lambda(\rho^\varepsilon)\nabla\cdot\left(\dfrac{\mathcal{M}^\varepsilon}{\rho^\varepsilon}\right)\right), & (1.5\text{b}) \\ \Delta\Phi^\varepsilon=\rho^\varepsilon. & (1.5\text{c}) \end{cases}$$