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A Local Inverse Conical Shock Problem for the Steady Supersonic Potential Flow

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Dedicated to Professor Gui-Qiang Chen on the occasion of his 60th birthday.

Abstract. This paper studies an inverse problem of reconstructing the shape of the circular symmetric cone for the given leading shock front. Assuming that the attack angle is small and that the incoming flow has a large Mach Number, we show that the shape of the cone can be reconstructed near the vertex from the data of the given shock and establish an asymptotic expansion for the velocity and the slope of cone.

AMS subject classifications: 35L67, 35L65, 76J20, 76K05, 76L05, 76M21

Key words: Inverse problem, steady supersonic potential flow, conical shock, cone.

1 Introduction

In Courant and Friedrichs famous book [9], for the steady supersonic flow moving at constant speed toward a straight circular cone with attack angle less than critical value, it is shown that there is a straight circular conical shock issuing from the vertex of wedge, ahead of which is the constant supersonic state and behind of which is a selfsimilar solution given by the apple curve. This shock is called the supersonic shock or transonic shock depending on the solution between the shock front and the cone. Such problem is called the direct prob-

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lem, and has been studied by many authors in various cases, see for instance, [1,3–8,10,11,16–18,26,29,37–39] and references therein.

In this paper, we are concerned with its inverse problem, which is to determine the shape of the cone for the given conical shock. Now we describe the problem as follows. The flow and the cone are assumed to be axial-symmetrical. Let x be the abscissa along the axis, y the distance from the axis, u and v the axial and radial components of velocity respectively, see Figs. 1 and 2. Then, the governed equations of the flow can be written as

$$\begin{cases} (\rho u)_x+(\rho v)_y+\dfrac{\rho v}{y}=0, \\ v_x-u_y=0, \\ \dfrac{1}{2}(u^2+v^2)+\dfrac{\gamma\rho^{\gamma-1}}{\gamma-1}=\dfrac{1}{2}(u_\infty^2)+\dfrac{c_\infty^2}{\gamma-1}. \end{cases} \tag{1.1}$$

Here $c(\rho)=\sqrt{\gamma\rho^{\gamma-1}}$ is the sonic speed, and the adiabatic exponent γ is a constant with $\gamma>1$, $(u_\infty,0)$ and ρ_∞ are the velocity and the density of incoming flow with

$$u_\infty>c(\rho_\infty)=c_\infty.$$

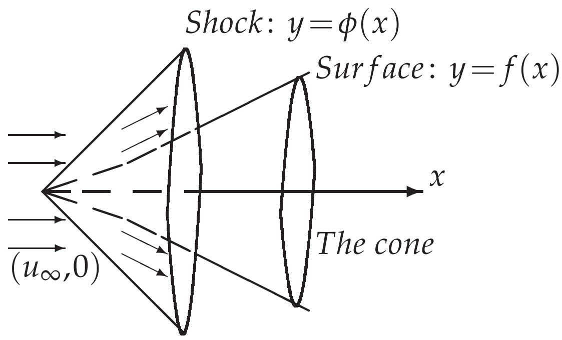

Figure 1: Supersonic conical flow.

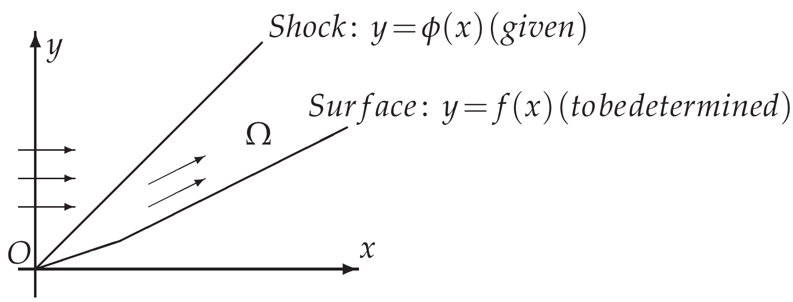

Figure 2: Supersonic conical flow.