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Local Well-Posedness of the Three Dimensional Linearized MHD Boundary Layer System

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Abstract. For the three dimensional linearized MHD boundary layer system around a shear flow, we establish a well-posedness result with Sobolev regularity in one tangential direction under the non-degeneracy condition in that direction. The proof is based on an observation of a new cancellation mechanism that is different from the two space dimensional case. Precisely, the new cancellation relies on the evolution of the magnetic field orthogonal to the boundary instead of the stream function in two space dimension. Even though this kind of cancellation can help to lower the regularity requirement in only one tangential direction while analyticity is still needed in the other tangential direction, we expect that this can be viewed as one step further to study the challenging problem on the well-posedness theory in Sobolev space of the three dimensional MHD boundary layer system under some suitable structural assumption.

AMS subject classifications: 76D10, 35G31

Key words: Well-posedness theory, cancellation mechanism, MHD boundary layer, structural assumptions.

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1 Introduction

This paper aims to investigate the existence and uniqueness of solutions to the three dimensional (3D) magnetohydrodynamic (MHD) boundary layer system, provided that the initial data are analytic in only one tangential variable. We use (u,v,w) and (f,g,h) to denote velocity and magnetic fields respectively, and denote by (x,y,z) the spatial variables in $\Omega=\{(x,y,z)\,|\,(x,y)\in\mathbb{T}^2,z>0\}$. Then the three-dimensional MHD boundary layer system in Ω reads

$$\begin{cases} \big(\partial_t+u\partial_x+v\partial_y+w\partial_z-\nu\partial_z^2\big)u-(f\partial_x+g\partial_y+h\partial_z)f+\partial_x p=0, \\ \big(\partial_t+u\partial_x+v\partial_y+w\partial_z-\nu\partial_z^2\big)v-(f\partial_x+g\partial_y+h\partial_z)g+\partial_y p=0, \\ \partial_t f+\partial_z(wf-uh)-\partial_y(ug-vf)-\mu\partial_z^2 f=0, \\ \partial_t g+\partial_x(ug-vf)-\partial_z(vh-wg)-\mu\partial_z^2 g=0, \\ \partial_t h+\partial_y(vh-wg)-\mu\partial_x(wf-uh)-\partial_z^2 h=0, \\ (u,v,\partial_y f,g)|_{y=0}=(0,0,0,0), \quad \lim\limits_{y\to+\infty}(u,v,f,g)=(U_\infty,V_\infty,F_\infty,G_\infty), \\ (u,v)|_{t=0}=(u_0,v_0), \qquad (f,g)|_{t=0}=(f_0,g_0), \end{cases} \tag{1.1}$$

where ν,μ are the viscosity and resistivity coefficients, respectively, and p,U_∞,V_∞, F_∞ and G_∞ are given functions of (t,x) variables satisfying the Bernoulli's law. In fact, these functions are the traces of the pressure, the tangential velocity and the tangential magnetic field in the ideal MHD system, respectively. To illustrate the cancellation mechanism in the linear level for shear flow, we may assume without loss of generality that $\mu=\nu\equiv 1$ and $(U_\infty,V_\infty,F_\infty,G_\infty)=(0,0,0,0)$ so that $\partial_x p=\partial_y p=0$ by the Bernoulli's law.

The derivation of the MHD boundary layer system (1.1) can be found in the paper Liu *et al.* [24] (see also the paper [7] by Gérard-Varet and Prestipino for the derivation with the insulating boundary condition on magnetic field). Note that by the boundary condition and divergence-free condition, we have

$$w(t,x,y,z)=-\int_0^z \big(\partial_x u(t,x,y,\tilde{z})+\partial_y v(t,x,y,\tilde{z})\big)\,d\tilde{z},$$
$$h(t,x,y,z)=-\int_0^z \big(\partial_x f(t,x,y,\tilde{z})+\partial_y g(t,x,y,\tilde{z})\big)\,d\tilde{z}.$$

These two nonlocal terms in the evolution of the tangential components of the velocity and magnetic fields give rise to the main difficulty in the well-posedness theory for (1.1) because they lead to loss of tangential derivatives.