Global Solutions to the Compressible Navier-Stokes Equations for a Reacting Mixture with Temperature Dependent Transport Coefficients

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Dedicated to Professor Gui-Qiang G. Chen on the occasion of his 60th birthday, with admiration and affection.

Abstract. We consider the compressible Navier-Stokes equations for a reacting ideal polytropic gas when the coefficients of viscosity, thermal conductivity, and species diffusion are general smooth functions of temperature. The choice of temperature-dependent transport coefficients is motivated by the kinetic theory and experimental results. We establish the existence, uniqueness, and time-asymptotic behavior of global solutions for one-dimensional, spherically symmetric, or cylindrically symmetric flows under certain assumptions on the H^2 norm of the initial data. This is a Nishida-Smoller type global solvability result, since the initial perturbations can be large if the adiabatic exponent is close to 1.

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1 Introduction

We study global well-posedness of a mathematical model governing the dynamic combustion of viscous and exothermically reacting gases with large initial data and temperature dependent transport coefficients. The motion of the gas can be described by the following compressible Navier-Stokes equations for a reacting mixture in the Eulerian coordinates (see Williams [56]):

$$
\begin{cases}
\rho_t + \nabla \cdot (\rho \boldsymbol{u}) = 0, \\
(\rho \boldsymbol{u})_t + \nabla \cdot (\rho \boldsymbol{u} \otimes \boldsymbol{u}) + \nabla P = \nabla \cdot \mathbb{S}, \\
(\rho E)_t + \nabla \cdot (\rho E \boldsymbol{u} + P \boldsymbol{u}) = \nabla \cdot (\kappa \nabla \theta + \mathbb{S} \boldsymbol{u} + q d \rho \nabla Z), \\
(\rho Z)_t + \nabla \cdot (\rho Z \boldsymbol{u}) = \nabla \cdot (d \rho \nabla Z) - K \phi(\theta) \rho Z.
\end{cases}
\tag{1.1}
$$

Here the density ρ, velocity $\boldsymbol{u} \in \mathbb{R}^n$, temperature θ, and reactant mass fraction $Z \in [0,1]$ are the primary unknowns of time $t \geqslant 0$ and spatial variable $\boldsymbol{x} \in \mathbb{R}^n$ with space dimension $n \geqslant 1$. The specific total energy E and viscous stress tensor $\mathbb{S}$ have the form

$$
E = e + \frac{1}{2}|\boldsymbol{u}|^2 + qZ, \quad \mathbb{S} = \mu\big(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^{\mathrm{T}}\big) + \lambda \nabla \cdot \boldsymbol{u} \, \mathbb{I}_n,
$$

where the constant q is the heat release, $\mu > 0$ and λ are the viscosity coefficients with $2\mu + n\lambda > 0$, and $\mathbb{I}_n$ is the identity matrix of order n. The pressure P, the internal energy e, and the transport coefficients κ (thermal conductivity), d (species diffusion), μ, and λ are prescribed through constitutive relations as functions of ρ and θ. The constant K is the rate of reactant, while $\phi(\theta)$ denotes the reaction rate function.

For ideal polytropic gases, the thermodynamic variables satisfy the equations of state

$$
e = c_v \theta, \quad P = R \rho \theta, \quad c_v = \frac{R}{\gamma - 1},
\tag{1.2}
$$

where $R > 0$ is the gas constant and $\gamma > 1$ is the adiabatic exponent. We assume that $\phi(\theta)$ is nonnegative and smooth for $\theta > 0$, which typically includes the modified Arrhenius equation $\phi(\theta) = \theta^\beta e^{-A/\theta}$ with β and $A > 0$ being constants [29].

Gui-Qiang Chen [2] initiated the study of global solutions to the reactive Navier-Stokes equations (1.1) in one dimension. Since then, global well-posedness for (1.1) has become an active topic of research (cf. [3,4,8,9,11,13–18,20,31, 33–36,39,42–45,50,51,54,55,57,58,60]). More precisely, for ideal polytropic gases (1.2), Chen [2] first established the global existence of large-data solutions to (1.1)