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Exploring Fractional-Order Nonlinear Dynamics in Biodiesel Production with Optimal Control

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Abstract. Biodiesel, a sustainable and renewable energy source, is a promising alternative to fossil fuels. The transesterification process of biodiesel production effectively captures memory effects in reaction kinetics. In this study, we developed two fractional order models of the chemical catalytic transesterification reaction to explore the memory effects of the reaction kinetics utilizing two different non-singular kernel methods: Caputo-Fabrizio and Atangana-Baleanu in the Caputo sense. We compared the results with experimental data of biodiesel production and demonstrated the existence and uniqueness of the solution for the fractional system. A sensitivity analysis is performed using the Latin hypercube sampling method to evaluate the impact of various parameters on biodiesel production, followed by the computation of partial rank correlation coefficients based on Pearson's correlation coefficient. We exhibit the dynamic behavior of all reactants corresponding to these fractional models with the variation of fractional order and the memory rate parameter. Additionally, we display the memory effect through the surface plots for biodiesel production by varying fractional order, molar ratio, and ultrasound frequency. Our numerical comparison with experimental data identifies the fractional-order value for the best fit of biodiesel production and can be increased by applying optimal control on ultrasound frequency.

AMS subject classifications: 34A08, 49-XX

Key words: Waste cooking oil, biodiesel, optimal control, Caputo-Fabrizio operator, Atangana-Baleanu in Caputo sense.

1 Introduction

Biodiesel is a leading alternative energy source to replace fossil fuels due to its favorable properties. It can be produced from various sources like vegetable oils, animal fats, mi-

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croorganisms, or waste cooking oil using a transesterification reaction catalyzed by either a base or an enzyme. Jatropha curcas and waste cooking oil are ideal for biodiesel production because they are non-food sources, reducing competition with food crops, and are cost-effective, with Jatropha thriving on marginal land and waste oil utilizing discarded resources. Waste cooking oil (WCO) is a by-product of various cooking processes in households, restaurants, food industries, and catering services. Improper disposal of waste cooking oil harms the environment by clogging drains, polluting water bodies, and releasing methane from landfills, contributing to climate change [11]. Reusing degraded oil in some food industries can increase harmful carcinogens like acrylamide and polycyclic aromatic hydrocarbons (PAHs), posing health risks such as cancer and cardiovascular diseases [13]. Despite its environmental and health concerns, waste cooking oil holds significant potential for sustainable applications. One of the most prominent uses of WCO is in the production of biodiesel, a renewable, eco-friendly alternative to fossil fuels [4, 21]. Biodiesel production from waste cooking oil contributes to a circular economy, where waste is turned into valuable products, ensuring sustainability and reducing overall waste generation [16]. It consists of triglycerides, but it often contains impurities such as water, free fatty acids (FFAs), food particles, and other organic matter. The transformation process primarily involves cleaning and preparing the oil for the transesterification reaction (see Fig. 1), which converts triglycerides into biodiesel [6].

Fractional-order derivatives have gained significant attention in mathematical modeling due to their ability to capture memory effects and hereditary properties in com-

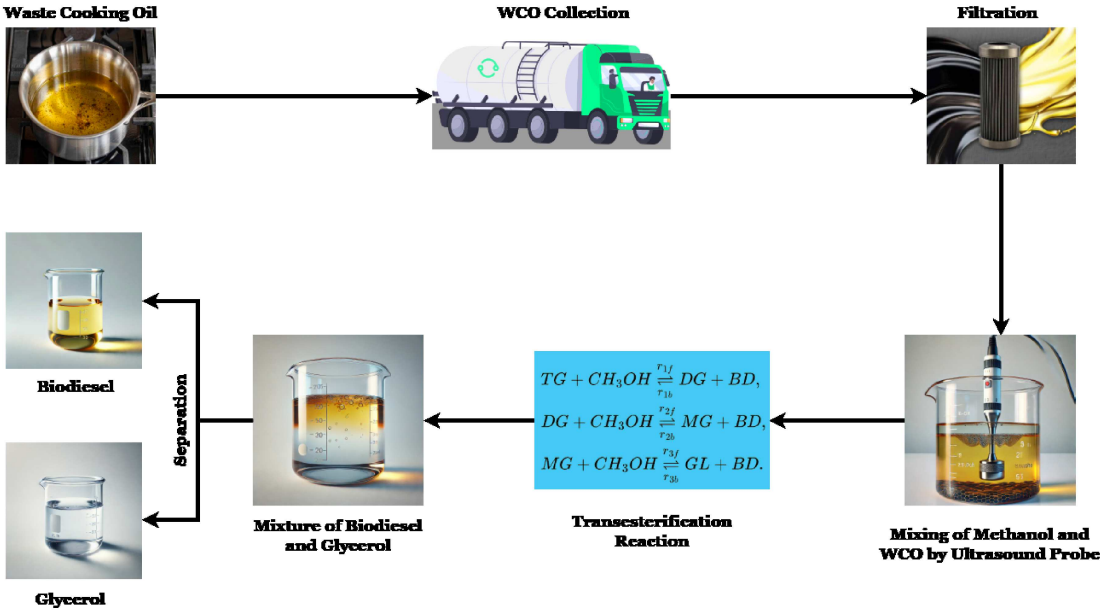

Figure 1: Scheme for WCO collection from various restaurants and households, filtration and transesterification reaction to produce biodiesel using ultrasound frequency.