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The Poisson-Garima Distribution: Additional Key Features and Its Significance on Statistical Process Control in Agriculture

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Abstract. This study primarily advances the theoretical development of the Poisson-Garima (PSNG) distribution by establishing several novel properties not previously addressed in existing literature. These theoretical enrichments enhance the model's statistical foundation and extend its applicability to overdispersed count data commonly observed in fields such as agriculture, biology, and medicine. As a secondary contribution, the improved PSNG model is applied to statistical process control through the development of PSNG-based control charts for monitoring count data. The proposed charts are evaluated via simulation studies and validated with an empirical agricultural dataset, where their performance is benchmarked against eight competing models. Additionally, comparisons between PSNG- and Poisson-based control charts demonstrate the superiority of the proposed approach in detecting process shifts under overdispersion. This integrated approach reinforces the PSNG distribution's theoretical depth while demonstrating its practical relevance in quality monitoring contexts.

AMS subject classifications: 62-xx, 62P30, 60Exx, 60E05, 62F10

Key words: Poisson-Garima distribution, numerical result, lifetime model, simulation study, statistical process control, real data study.

1 Introduction

The analysis of count data is fundamental in many real-world applications, especially in quality control, biostatistics, and industrial engineering. In particular, statistical process

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control (SPC) plays a vital role in monitoring and improving the quality of manufacturing processes [15, 36, 37]. Control charts are a primary tool in SPC, allowing for real-time detection of deviations from a stable process. Among these, the c-chart, based on the Poisson (PSN) distribution, is widely used to model the number of nonconformities per unit.

However, a well-known limitation of the PSN model is its equidispersion assumption – that the mean and variance are equal. In practice, this condition is often violated, especially in biological, agricultural, and industrial processes, where over-dispersion is frequently observed [17, 23, 26, 33–35].

To address this, several PSN-mixed models have been proposed, including the such as PSN-Lindley distribution [18], the PSN-Weibull distribution [12], the PSN-Lindley distribution [9], generalize PSN-Lindley distribution [30], the PSN-transmuted exponential distribution [19], the size biased PSN-Ishita distribution [25], the PSN-Xgamma distribution [31], the PSN-generalized Rayleigh distribution [24], the PSN-XLindley distribution [16], the PSN-Mirra distribution [3], PSN-moment exponential distribution [21] and the uniform PSN-Ailamujia [2], discrete Burr-Hatke [5], PSN-Ram Awadh [13], PSN-Ramos-Louzada (PSNRL, [32]), discrete moment exponential (DMNTEx, [6]), discrete Bilal (DBlal, [1]), PSN entropy-based weighted exponential [8], and Poisson-XRani distributions [7].

Among these, the Poisson-Garima distribution, introduced by Shanker [29], offers improved flexibility by assuming the Poisson rate parameter follows the Garima distribution. Where the probability density function (PDF) associated with Garima distribution [28] is defined as follows:

$$f(y;\eta)=\frac{\eta}{\eta+2}(1+\eta+\eta y)e^{-\eta y}, \quad \eta>0, \quad y>0.$$

The PSNG model has been shown to perform well in over-dispersed settings and has proven competitive in modeling real-world count data. Although the PSNG distribution has already been introduced in prior literature, this study contributes by developing new theoretical properties and statistical measures of the PSNG model that were not addressed in earlier work. Specifically, we establish novel results for the reliability characteristics, cumulative distribution function, mean deviation, Lorenz curve, stress – strength reliability, and entropy measures, which enhance the model's utility for data analysis and modeling. These developments justify a concise review of the PSNG distribution to establish a integrated foundation for the new results.

While the paper includes an SPC application based on the PSNG distribution namely, the construction of a Shewhart-type control chart for over-dispersed count data [22], the primary objective of this study lies in the distributional theory and novel characterization of the PSNG model. The control chart development is included as a supporting application to demonstrate the practical relevance of the proposed theoretical advancements. This study is primarily devoted to advancing the theoretical framework of the PSNG distribution by introducing several novel properties that have not been previously explored