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Optimizing Interdisciplinarity in Data Analysis with the Adapted Alpha-Power Transformation of the Nadarajah-Haghighi Distribution

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Abstract. The practice of incorporating extra parameters into standard models is a common technique in statistical analysis. Adding an extra parameter enables the formation of a new model by applying the modified alpha-power transformation, employing the Nadarajah-Haghighi model as the baseline. Numerous characteristics of the said model are acquired, like the mode, quantiles, entropies, stochastic orders, mean residual life function, and order statistics. The maximum likelihood estimation method has been employed to estimate the parameters of the suggested model. To show how well the suggested distributions will function in a real-world setting, a simulation study has also been carried out and data has been examined. It is attained that the proposed model outperforms several other cutting-edge, current models as well as the baseline.

AMS subject classifications: 62-xx, 60Exx, 60E05, 62F10
Key words: Modified alpha-power transformation, Nadarajah-Haghighi, characteristic function, simulation study, real data sets.

1 Introduction

Probability distributions serve as a foundational tool in decision-making under conditions of uncertainty. They are employed in engineering, signal processing, communi-

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cation systems, survival analysis, and reliability analysis, among other fields. Upgrading over regular distributions has been an accepted method in statistical theory during the last few decades [15, 37–39]. This adjustment intends to make the classical distributions more tractable so that complicated data structures can be usefully analyzed. The already existing probability distribution is insufficient to represent non-monotonic hazard function data, according to the literature on probability theory. One example of a non-monotonic hazard shape that cannot be described by the Weibull distribution [42] is bathtub data. To address this issue, researchers are attempting to modify the present distributions. Modifications are implemented by either adding the number of parameters in the baseline model or developing novel approaches for generating probability distributions. Aldeni *et al.* [3] designed a novel distribution family, for instance, created on quantile of a generalized lambda model. T-normal family of distributions was explored by Alzaatreh *et al.* [5]. Alzaatreh *et al.* [6] presented the generalized Cauchy family of models. The half-Cauchy family of distributions was presented by Cordeiro *et al.* [11] and applied to actual events. Weibull-R family of distribution features and applications was studied by Ghosh and Nadarajah [16].

More recently, [26] introduced an entirely novel method for the addition of an extra parameter to a continuous model termed alpha power transformation (APT). Essentially, the concept was presented to add skewness to a baseline model. In particular, the generator was utilized to convert an exponential distribution with one parameter into an alpha-power exponential distribution with two parameters. Explicit formulas for the attributes of distribution, and entropies were among the features of the proposed model that were examined. The aforementioned generator has been effectively utilized by [15, 30] to convert a two-parameter Weibull model into a three-parameter alpha power Weibull model. Many researchers have used the transformation to derive distributions with transformed alpha power, such as the transformed inverse Lindly distribution [13], the transformed extended exponential model [17], the transformed Lindly distribution [14], and the transformed generalized exponential model [12].

The Gull alpha power Weibull model was concentrated on more recently by Ijaz *et al.* [20], who also gave the unique family of models [19,21] detailed the improvements made to the Lomax model.

The Nadarajah-Haghighi (NHh) model, which has two parameters and is an extension of an exponential model as an alternative to Weibull, exponentiated exponential (EE) and gamma models, was presented and investigated in [29]. NHh may occasionally not be able to adequately fit the data due to its failure rate function, which can be growing, declining, or constant. In practice, there are several ways to project risks and losses; for instance, human mortality and the life cycle of a machine have more variable behavior. For this reason, many modifications and extensions of the NHh model with varying numbers of parameters were proposed by researchers, for example, [10, 23, 27, 31, 40].

The purpose of this work is to get a suitable fit by focusing on the novel family of probability models and deriving its particular form using the cumulative distribution function (CDF) of NHh model. The next part examines several modified alpha power