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A Mathematical Perspective on Corruption's Effect on Productivity and Service Efficiency with Cost-Effective Control Strategies

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Abstract. Corruption remains a major barrier to development in Africa, undermining public trust, weakening service delivery, and hindering economic growth. Defined as the abuse of public power for personal gain, it appears in forms such as bribery, embezzlement, nepotism, and cronyism, affecting both public and private sectors. Despite various anti-corruption efforts, the problem persists. This study presents a non-linear system of differential equations to evaluate the impact of corruption on productivity and service delivery, incorporating both constant and time-dependent control strategies. The basic reproduction number (R_0) is calculated using the next-generation matrix method. Model behaviour is further analysed through sensitivity analysis and stability assessments using the Routh-Hurwitz criterion and Lyapunov functions. An optimal control framework, based on Pontryagin's maximum principle, assesses the effectiveness and cost-efficiency of preventative and punitive interventions. Numerical simulations, using the fourth-order Runge-Kutta method, indicate that strong enforcement of punitive measures is the most effective and cost-efficient strategy to combat corruption. These measures significantly improve productivity and public service delivery. The key policy recommendation is that governments and institutions should prioritize robust punitive controls as the primary approach to reducing corruption and promoting sustainable development.

AMS subject classifications: 37N99, 37N40

Key words: Corruption dynamics, productivity and service delivery, basic reproductive number, sensitivity analysis, optimal control analysis, cost-effectiveness analysis, numerical simulation.

1 Introduction

Corruption deprives communities of essential opportunities, including employment, efficient service delivery, and productivity, particularly in critical institutions like hospi-

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tals and police stations. The word "corrupt" originates from the Latin verb corrumpere, meaning to shatter or ruin [20]. Corruption is a criminal act involving bribery, fraud, financial crimes, favoritism, nepotism, and abuse of power by public or private officials for personal gain. It arises from various factors such as unemployment, poverty, inadequate education, greed, limited civic engagement, and weak political transparency [15, 22]. For instance, financial hardship can drive individuals to accept bribes for survival, while greed and peer pressure further normalize corruption. This erosion of integrity weakens government institutions, reduces public trust, and hampers their ability to address crises and service delivery issues. Moreover, corruption discourages investment, stifles economic growth, and exacerbates poverty. Poor service delivery – the government's distribution of essential resources like water, sanitation, electricity, and housing – is a widespread challenge, leading to declining resources, high unemployment, rising crime, and deteriorating living conditions [5]. This discourages business investments and further harms the economy. Despite anti-corruption laws in every country, corruption remains pervasive, ultimately reducing productivity, worsening service delivery, and slowing economic growth [10].

To effectively combat corruption, mathematical modelling provides valuable insights for policymakers, helping them understand corruption dynamics and optimize resource allocation at minimal cost. Several researchers have explored corruption through mathematical models. Gweryina *et al.* [12] examined corruption in Nigeria, emphasizing the impact of an immunity clause. Their study suggested that removing this clause, combined with strict control measures like convictions and the death penalty, could significantly reduce corruption. Baba *et al.* [4] developed a fractional-order model to analyze the relationship between terrorism and corruption, revealing that both initially increase together but eventually decline as public interest wanes. Eegunjobi and Makinde [9] introduced a nonlinear mathematical model to assess how honest individuals become exposed to corrupt practices, using an SECRH compartmental framework. These studies highlight the effectiveness of mathematical modelling in understanding and addressing corruption.

Optimal control analysis plays a crucial role in identifying the most effective strategies to combat corruption, boost productivity, and enhance service delivery. Incorporating cost-effectiveness analysis through the incremental cost-effectiveness ratio (ICER) enables researchers to identify the most cost-effective measures, allowing policymakers to allocate resources efficiently. Athithan *et al.* [3] applied the SIR model to analyze corruption evolution, highlighting that increased media coverage and stricter punishments effectively reduce corruption levels. Fantaye and Birhanu [11] developed a deterministic model incorporating societal influence on honest individuals and examined optimal control strategies, including education via social media and press, anti-corruption laws, and punitive measures. Their results showed that a combined approach of prevention and punishment is the most effective. Islam *et al.* [13] used a criminological model with four compartments to demonstrate that preventing corrupt situations while punishing offenders significantly lowers corruption. Jose *et al.* [14] investigated mathematical modelling