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DeepSPIM: Deep Semi-Proximal Iterative Method for Sparse-View CT Reconstruction with Convergence Guarantee

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Abstract. Computed tomography (CT) reconstruction with sparse-view projections is a challenging problem in medical imaging. The learning-based methods lack generalization ability and mathematical interpretability. Since the model-based iterative reconstruction (IR) methods need inner gradient-based iterations to deal with the CT system matrix, the algorithms may not be efficient enough, and IR methods with deep networks have no convergence guarantees. In this paper, we propose an efficient deep semi-proximal iterative method (DeepSPIM) to reconstruct CT images from sparse-view projections. Unlike the existing IR methods, a carefully designed semi-proximal term is introduced to make the system matrix-related subproblem solvable. Theoretically, we give some useful mathematical analysis, including the existence of the solutions to the reconstruction model with an implicit image prior, the global convergence of the proposed method under gradient step denoiser assumption. Experimental results show that DeepSPIM is efficient and outperforms the closely related state-of-the-art methods regarding quantitative image quality values, details preservation, and structure recovery.

AMS subject classifications: 94A08, 68U10

Key words: Computed tomography, sparse-view reconstruction, iterative method, semi-proximal term, global convergence.

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1 Introduction

Over the past few decades, X-ray computed tomography (CT) has been one of the most important diagnostic imaging techniques. However, radiation exposure may cause genetic mutations and increase cancer risk [8]. Low-dose CT can significantly reduce the threat of X-ray radiation. Sparse-view CT is a major strategy that only utilizes a few projected views for image reconstruction [10,39,40]. Unfortunately, the sparse-view CT strategy can also compromise image quality [6]. As a result, numerous exciting and practical models and algorithms have been proposed. In this paper, we focus on the semi-proximal iterative method for sparse-view CT reconstruction.

The following linear system can express the forward model for CT imaging:

$$f = Ru + \xi, \tag{1.1}$$

where $f \in \mathbb{R}^m$ is a vectorized projected data, $u \in \mathbb{R}^n$ is a vectorized 2D image, $R \in \mathbb{R}^{m\times n}$ is the projection matrix, and ξ is the Gaussian white noise added to the projected data. Due to the ill-posedness of the sparse-view CT image reconstruction problem, it is challenging to reconstruct u from f in practice. The most widely used method is the filtered back-projection (FBP) algorithm proposed by Kak and Slaney [28]. FBP is an efficient and robust direct reconstruction method. However, when the projection views are limited and polluted, FBP suffers from streaking artifacts. To address this issue, various methods have been proposed.

The first category is iterative reconstruction (IR) methods. Gordon *et al.* [19] presented the first IR method, the algebraic reconstruction technique (ART) to reconstruct CT images iteratively from no more than 60 views. Andersen and Kak [2] improved ART by applying the error correction terms simultaneously and presented the simultaneous algebraic reconstruction technique (SART). Shepp and Vardi [15] proposed the expectation maximization (EM) method to maximize the probability of the potential signal given the projected data. Though these methods are computationally efficient, it is difficult to provide satisfactory results when the projection views are highly sparse without image priors. In order to characterize the image features, Sidky *et al.* [39] applied the total variation (TV) prior to the few views and limited-angle data in divergent-beam CT. Sidky and Pan [40] have also studied the constrained TV minimization method for cone-beam CT. In order to design an efficient algorithmic for total variation based image restoration, Chen *et al.* [12] proposed a primal-dual fixed point algorithm for CT reconstruction. Kim *et al.* [31] improved the reconstruction effects by non-local TV prior. Xu *et al.* [51] introduced the dictionary-learning-based method to low-dose CT reconstruction. An improved tensor dictionary learning method is proposed by Wu *et al.* [48] for low-dose spectral CT reconstruction. IR methods with learning-based priors have also been studied. As an important method in compressed sensing, the convolutional sparse coding prior with gradient regularization (CSCGR) has been presented by Bao *et al.* [3] for sparse-view CT reconstruction. By directly working on the whole image, CSCGR can maintain