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Total Variations for Hue, Saturation, and Value of a Color Image

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Abstract. In this paper, we develop total variations for hue, saturation, and value of a color image, and we propose a novel hue-saturation-value total variation model for color image restoration. We first refine the hue formulation of a color image to make it mathematically and applicationally meaningful by assigning different hue values to different colors. We then develop the proposed hue-saturation-value total variation based on the conception of hue/saturation/value gradient. We investigate the dual formulation and the properties of the proposed hue-saturation-value total variation, and we finally propose a color image restoration model which is formulated by combing the proposed hue-saturation-value total variation regularization with the data-fitting term between the objective color image and the observed color image. We develop an efficient alternating iterative algorithm to solve the proposed optimization model in practice, and we give the convergence analysis of the proposed algorithm. Numerical examples are presented to demonstrate that the performance of the proposed hue-saturation-value total variation and the proposed color restoration model is better than that of other testing methods in terms of visual quality, peak signal-to-noise ratio (PSNR), structural similarity index (SSIM), and S-CIELAB color error.

AMS subject classifications: 65K10, 65J22, 68U10, 90C26

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1 Introduction

The representation ways of color images have been extensively developed [5]. One of the most popular color representation models is RGB (red, green, blue) color space, which

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is the most general hardware-oriented model. Another widely used color space is HSV (hue, saturation, value) color space, which is similar to human color perception. In the literature, many color image processing models are proposed based on different color spaces, e.g. color image restoration [2,4,9,13,28,30,36], color image enhancement [9,15] based on RGB color space, color image restoration [8], color image segmentation [10,20, 39] based on HSV color space, color image restoration [22], color image enhancement [1] based on CIELAB color space, color image enhancement [33], color image restoration [19, 21,23,25,32,34], color image decomposition [26,37] based on opponent color space, etc.

Images will be affected by the interference of imaging equipment and external environment noise during digitizing and transmitting. Therefore, regularization methods need to be designed to deal with image restoration problem. As one of the most popular regularization methods, total variation (TV) [29] is proposed to deal with grayscale images, and has been developed into many other forms for handling corresponding image processing problems. For instance, anisotropic TV [14] is designed for image decomposition problem, and nonlocal TV [16,31] makes use of image structures and features for image restoration problem. On the other hand, TV regularization is also generalized for vector-valued (color of multichannel) image regularization. Bresson and Chan [4] proposed a color TV regularization with a local channel-coupling

$$\text{C-TV}(\mathbf{u}):=\int_\Omega \sqrt{\sum_{k=1}^{c}\big(\partial_x u_k(x,y)\big)^2+\big(\partial_y u_k(x,y)\big)^2}dxdy,$$

where Ω is the image domain, c is the number of channels, $u_k(x,y)$ is the k-th channel of vector-valued image $\mathbf{u}=[u_1(x,y),u_2(x,y),\cdots,u_c(x,y)]^\intercal$ and $\partial_x u_k$ (or $\partial_y u_k$) is the partial derivative of u_k with respect to x (or y). In [13], the comparison of the TV regularization with the local-channel coupling C-TV and the global channel coupling C-TV has been studied, which shows that C-TV can determine edge locations in different channels. Moreover, Duran *et al.* [13] studied the gradient of a multispectral image as a three-dimensional (3D) matrix or tensor with the dimensions corresponding to the spatial and spectral channels. Different norms along different dimensions can be employed to measure the smoothness of this tensor. Paul *et al.* [28] proposed the generalized vector-valued total variation (GV-TV) by coupling different channels with different norms. We remark here that many efficient algorithms can be applied to solve different vectorial total variation models, such as the augmented Lagrangian method [18,27,38], dual method [6,7,38], the split Bregman algorithm [3,38] and the iteratively reweighted norm algorithm [28].

Color images can be studied from the view of Riemann geometry. In [12], a vector-valued image was considered as a parametric 2D manifold embedded in a C-dimensional space, where C is the number of channels. One idea is to study the spectral information of $(\nabla\mathbf{u}(x,y))^\intercal\nabla\mathbf{u}(x,y)$, which refers to the structure tensor of the image. The spectral information would be useful for describing edges of color images. Sapiro [30] proposed the following vectorial TV model:

$$\text{V-TV}(\mathbf{u}):=\int_\Omega \varphi\big(\lambda^+(x,y),\lambda^-(x,y)\big)dxdy,$$