CSIAM Trans. Life Sci.
doi: 10.4208/csiam-ls.SO-2025-0021

Vol. **x**, No. x, pp. 1-24
xx 2025

On a Juvenile-Adult Model: The Effects of Seasonal Succession and Harvesting Pulse

Haiyan Xu[1], Zhigui Lin[1,*] and Michael Pedersen[2]

[1] *School of Mathematics, Yangzhou University, Yangzhou 225002, China.*
[2] *Department of Applied Mathematics and Computer Science, Technical University of Denmark, DK 2800, Lyngby, Denmark.*

Received 2 July 2025; Accepted 7 October 2025

Abstract. In this paper, a juvenile-adult population model incorporating seasonal succession and pulsed harvesting is developed. The seasonal succession captures the cyclical change between favorable and unfavorable environmental conditions, while the pulsed harvesting represents a periodic human intervention, targeting the adult population exclusively during favorable seasons. The principal eigenvalue for the corresponding linearized system is defined and its dependence on both the intensity of the harvesting pulses and the duration of the unfavorable season is analyzed. Explicit expressions and analysis of the principal eigenvalue for a logistic model extended with seasonal succession and pulsed harvesting are provided specifically. Based on the principal eigenvalue, we establish sufficient conditions for population persistence and extinction. Numerical simulations are conducted to validate these analytical results. Our findings demonstrate that higher harvesting intensity during the favorable season is detrimental to species survival. Furthermore, extending the duration of the unfavorable season can trigger a critical transition from population persistence to extinction.

AMS subject classifications: 35R12, 35R35, 92D25

Key words: Age-structured model, seasonal succession, harvesting pulse, persistence, extinction.

1 Introduction

This paper presents a juvenile-adult model with seasonal succession and harvesting pulses exerting on adults in favorable seasons, of the following form:

*Corresponding author. *Email addresses:* zglin@yzu.edu.cn (Z. Lin), hyanxu0829@163.com (H. Xu), micp@dtu.dk (M. Pedersen)

$$
\begin{cases}
u_{1t}-d_1u_{1xx}=bu_2-(a+m_1)u_1-\alpha_1u_1^2, & t\in\Omega^r_{n,\rho}, \quad x\in(-L,L), \quad (1.1a)\\
u_{2t}-d_2u_{2xx}=au_1-m_2u_2-\alpha_2u_2^2, & t\in\Omega^r_{n,\rho}, \quad x\in(-L,L), \quad (1.1b)\\
u_{1t}=-k_1u_1, & t\in\Omega^l_n, \quad x\in(-L,L), \quad (1.1c)\\
u_{2t}=-k_2u_2, & t\in\Omega^l_n, \quad x\in(-L,L), \quad (1.1d)\\
u_1\big((n\tau+(1-\delta)\rho\tau)^+,x\big)=u_1\big(n\tau+(1-\delta)\rho\tau,x\big), & x\in(-L,L), \quad (1.1e)\\
u_2\big((n\tau+(1-\delta)\rho\tau)^+,x\big)=hu_2\big(n\tau+(1-\delta)\rho\tau,x\big), & x\in(-L,L), \quad (1.1f)\\
u_i(t,-L)=u_i(t,L)=0, & t\in(0,+\infty), \quad (1.1g)\\
u_i(0,x)=u_{i,0}(x), \quad i=1,2, & x\in[-L,L], \quad (1.1h)
\end{cases}
$$

where, for $n=0,1,2,\ldots,$

$$
\Omega^r_{n,\rho}=\big(n\tau,n\tau+(1-\delta)\rho\tau\big]\bigcup\big((n\tau+(1-\delta)\rho\tau)^+,n\tau+(1-\delta)\tau\big],
$$
$$
\Omega^l_n=\big(n\tau+(1-\delta)\tau,(n+1)\tau\big]
$$

with $\rho,\delta\in(0,1)$, and harvesting coefficient $h\in(0,1]$, so $(1-h)\in[0,1)$ is naturally used to characterize the harvesting rate on adults. The unknown $u_1(t,x)$ and $u_2(t,x)$ are the densities of juveniles and adults, respectively, and the positive constants d_1 and d_2 are the diffusive rates of juveniles and adults, respectively. b denotes the reproduction rate of adults and a is the rate at which juveniles mature into adults. m_1 and m_2 represent the death rates of juveniles and adults. α_1 and α_2 denote the competition coefficients of juvenile and adult individuals, respectively. Biologically, species experiences two different seasons, one favorable and one unfavorable, which we call the good season (warm season) and the bad season (cold season), in accordance with standard literature. $\Omega^r_{n,\rho}$ can be regarded as a good season (or warm days) that is beneficial for species diffusion and the species development is governed by a logistic equation, while Ω^l_n is a bad season arising from limited resources such as food and cold weather that cause a decline in the species density. Here the species development is governed by a Malthusian equation, where positive constants k_1 and k_2 are the mortality rates of juveniles and adults in the cold season, respectively. $u_2((n\tau+(1-\delta)\rho\tau)^+,x)$ with $0<\rho<1$ describes the density of the adults after being harvested at time $t=n\tau+(1-\delta)\rho\tau$ in the good season. To make it clear, we here present Fig. 1 to show the impact of the harvesting pulse and seasonal succession on adults.

It is widely recognized that environmental fluctuations over time significantly influence species' growth and developmental processes [1, 4, 8, 12]. Notably, seasonal variations not only alter species growth patterns but also play a pivotal role in shaping community structures. A case of this is observed in temperate lakes, where phytoplankton and zooplankton thrive during warmer months before entering dormancy or perishing in winter [2, 7].

To explore how seasonal succession impacts population dynamics, researchers such as Steiner *et al.* [21], Hu and Tessier [9] conducted extensive experiments, gathering data

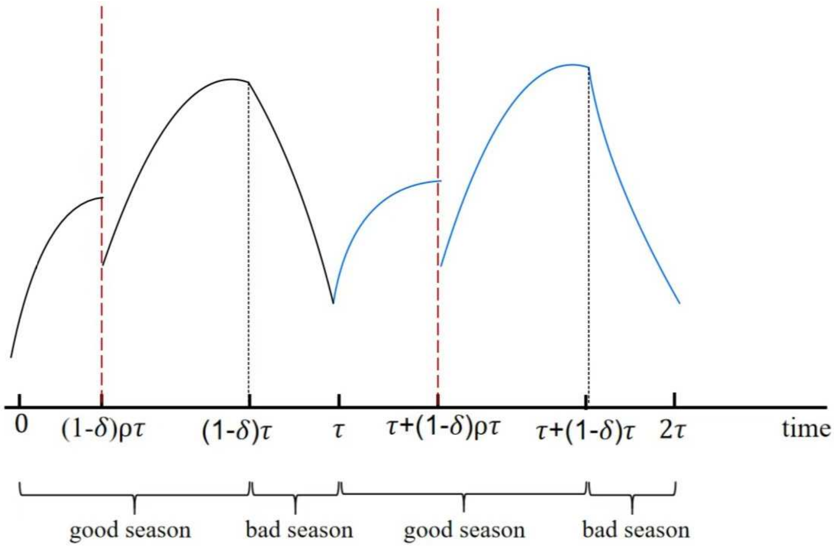


Figure 1: A diagram showing harvesting on adults with seasonal succession.

on phytoplankton competition under seasonal changes. Meanwhile, Klausmeier [10] analyzed the Rosenzweig-McArthur model to examine seasonal effects on its dynamic behavior. Subsequently, Hsu and Zhao [8] provided a comprehensive investigation into the dynamics of a class of Lotka-Volterra competitive models, incorporating seasonal succession.

By integrating seasonal succession into a diffusive logistic model with a free boundary, Peng and Zhao [17] established critical thresholds for species spread or extinction, determining whether a species expands indefinitely or disappears within a finite range. Their work also highlighted how the durations of warm and cold seasons influence ecological dynamics. Additionally, they quantified the spreading speed of species under successful expansion scenarios. Inspired by the works in [17], the authors in [18] considered a West Nile virus nonlocal model with free boundaries and seasonal succession. Their results show that the spreading or vanishing of the virus depends not only on the initial infection length, the initial infection scale and the spreading ability to new areas, but also on the duration of the warm season.

The dynamics of populations are influenced not only by seasons but also by external human interventions [13, 23, 25, 26]. Impulsive effects in the form of human intervention are widely considered in population ecology. We refer to a predator-prey system in [20] for the conservation of biological diversity, pest management system [22] for vegetation protection, control system [6] for optimal placement and planting in fishery and agriculture, and [24] for an invasive system with nonlocal dispersal to sustainable development. See also different pulse interventions in an epidemic model in [3, 5, 19].

The rest of this paper is arranged as follows. Section 2 presents the existence and properties of the principal eigenvalue, then an example of a logistic model with harvesting pulse and seasonal succession is introduced and investigated. The global existence of the solution is shown in Section 3, and the dynamics of the solution is discussed. Numerical approximations involving harvesting intensity and duration of seasons are finally exhibited in Section 4.

2 The principal eigenvalue

To analyze the principal eigenvalue problem, we first consider the corresponding periodic problem of problem (1.1)

$$\begin{cases} U_t - d_1 U_{xx} = bV - (a+m_1)U - \alpha_1 U^2, & t \in \Omega^r_{0,\rho}, \quad x \in (-L,L), \\ V_t - d_2 V_{xx} = aU - m_2 V - \alpha_2 V^2, & t \in \Omega^r_{0,\rho}, \quad x \in (-L,L), \\ U_t = -k_1 U, & t \in \Omega^l_0, \quad x \in (-L,L), \\ V_t = -k_2 V, & t \in \Omega^l_0, \quad x \in (-L,L), \\ U\big(((1-\delta)\rho\tau)^+, x\big) = U\big((1-\delta)\rho\tau, x\big), & x \in (-L,L), \\ V\big(((1-\delta)\rho\tau)^+, x\big) = hV\big((1-\delta)\rho\tau, x\big), & x \in (-L,L), \\ U(t,\pm L) = V(t,\pm L) = 0, & t \in [0,\tau], \\ U(0,x) = U(\tau,x), \quad V(0,x) = V(\tau,x), & x \in [-L,L]. \end{cases} \tag{2.1}$$

Its solution depends on the principal eigenvalue of the following periodic eigenvalue problem:

$$\begin{cases} \phi_t - d_1 \phi_{xx} = b\psi - (a+m_1)\phi + \lambda\phi, & t \in \Omega^r_{0,\rho}, \quad x \in (-L,L), & \text{(2.2a)} \\ \psi_t - d_2 \psi_{xx} = a\phi - m_2 \psi + \lambda\psi, & t \in \Omega^r_{0,\rho}, \quad x \in (-L,L), & \text{(2.2b)} \\ \phi_t = -k_1 \phi + \lambda\phi, & t \in \Omega^l_0, \quad x \in (-L,L), & \text{(2.2c)} \\ \psi_t = -k_2 \psi + \lambda\psi, & t \in \Omega^l_0, \quad x \in (-L,L), & \text{(2.2d)} \\ \phi\big(((1-\delta)\rho\tau)^+, x\big) = \phi\big((1-\delta)\rho\tau, x\big), & x \in (-L,L), & \text{(2.2e)} \\ \psi\big(((1-\delta)\rho\tau)^+, x\big) = h\psi\big((1-\delta)\rho\tau, x\big), & x \in (-L,L), & \text{(2.2f)} \\ \phi(t,\pm L) = \psi(t,\pm L) = 0, & t \in [0,\tau], & \text{(2.2g)} \\ \phi(0,x) = \phi(\tau,x), \quad \psi(0,x) = \psi(\tau,x), & x \in [-L,L]. & \text{(2.2h)} \end{cases}$$

In problem (2.2), let

$$\phi(t,x) = f_1(t)\Psi(x), \quad \psi(t,x) = f_2(t)\Psi(x)$$

with Ψ satisfying

$$\begin{cases} -\Psi_{xx} = \lambda_0 \Psi, & x \in (-L,L), \\ \Psi(-L) = \Psi(L) = 0. \end{cases}$$

Then problem (2.2) can be written as

$$\begin{cases} f_1'(t)=bf_2-(a+m_1+d_1\lambda_0)f_1+\lambda f_1(t), & t\in\Omega_{0,\rho}^r, & (2.3a)\\ f_2'(t)=af_1-(m_2+d_2\lambda_0)f_2+\lambda f_2, & t\in\Omega_{0,\rho}^r, & (2.3b)\\ f_1\big((1-\delta)\tau\big)=f_1(0)e^{(k_1-\lambda)\delta\tau}, & t\in\Omega_0^l, & (2.3c)\\ f_2\big((1-\delta)\tau\big)=f_2(0)e^{(k_2-\lambda)\delta\tau}, & t\in\Omega_0^l, & (2.3d)\\ f_1\big(((1-\delta)\rho\tau)^+\big)=f_1\big((1-\delta)\rho\tau\big), & & (2.3e)\\ f_2\big(((1-\delta)\rho\tau)^+\big)=hf_2\big((1-\delta)\rho\tau\big), & & (2.3f)\\ f_1(0)=f_1(\tau),\quad f_2(0)=f_2(\tau). & & (2.3g)\end{cases}$$

For the Eqs. (2.3a), (2.3b) and pulse conditions in (2.3), it then follows from [25], by direct calculations, that $(f_1(t),f_2(t))$ satisfies

$$\begin{cases} f_1(t)=\dfrac{N_1be^{\mu_1t}+N_2(m_2+d_2\lambda_0+c_2)e^{\mu_2t}}{C}, & t\in(0,(1-\delta)\tau],\\ f_2(t)=\dfrac{-N_1(m_2+d_2\lambda_0+c_2)e^{\mu_1t}+N_2ae^{\mu_2t}}{C}, & t\in\big(0,(1-\delta)\rho\tau\big],\\ f_2(t)=\dfrac{-hN_1(m_2+d_2\lambda_0+c_2)e^{\mu_1t}+hN_2ae^{\mu_2t}}{C}, & t\in\big(((1-\delta)\rho\tau)^+,(1-\delta)\tau\big],\end{cases} \tag{2.4}$$

where $\mu_{1,2}=\lambda+c_{1,2}$ with

$$c_{1,2}=\frac{-[a+m_1+m_2+(d_1+d_2)\lambda_0]\pm\sqrt{(a+m_1-m_2+d_1\lambda_0-d_2\lambda_0)^2+4ab}}{2}.$$

Without loss of generality, assume that $c_1>c_2$, then

$$\begin{aligned}&a+m_1+d_1\lambda_0+c_1=-(m_2+d_2\lambda_0+c_2)>0,\\ &C:=ab-(a+m_1+d_1\lambda_0+c_1)(m_2+d_2\lambda_0+c_2)>0.\end{aligned}$$

Denote $A:=a+m_1+d_1\lambda_0+c_1(>0)$, then substituting the Eqs. (2.3c) and (2.3d) into (2.4), yields

$$\begin{cases} N_1be^{\mu_1(1-\delta)\tau}-AN_2e^{\mu_2(1-\delta)\tau}=(N_1b-AN_2)e^{(k_1-\lambda)\delta\tau},\\ hN_1Ae^{\mu_1(1-\delta)\tau}+hN_2ae^{\mu_2(1-\delta)\tau}=(N_1A+N_2a)e^{(k_2-\lambda)\delta\tau}.\end{cases} \tag{2.5}$$

For abbreviation, we further denote $N_1=1,\Lambda=e^{\lambda\tau}$, and

$$\begin{aligned}&A_{11}=Ae^{c_2(1-\delta)\tau}, && A_{12}=be^{k_1\delta\tau}, && A_{13}=be^{c_1(1-\delta)\tau}, && A_{14}=Ae^{k_1\delta\tau},\\ &A_{21}=Ae^{k_2\delta\tau}, && A_{22}=hAe^{c_1(1-\delta)\tau}, && A_{23}=hae^{c_2(1-\delta)\tau}, && A_{24}=ae^{k_2\delta\tau}.\end{aligned}$$

Then (2.5) can be written as

$$\begin{cases} A_{13}\Lambda - A_{11}N_2\Lambda = A_{12} - A_{14}N_2, \\ A_{22}\Lambda + A_{23}N_2\Lambda = A_{21} + A_{24}N_2, \end{cases}$$

which is equivalent to

$$\begin{cases} \Lambda = \dfrac{A_{12} - A_{14}N_2}{A_{13} - A_{11}N_2}, \\ \Lambda = \dfrac{A_{21} + A_{24}N_2}{A_{22} + A_{23}N_2}. \end{cases} \tag{2.6}$$

One easily checks from the first function (N_2,Λ) in (2.6) that it passes through two fixed points $P_1(b/A,0)$ and $P_2(0,e^{k_1\delta\tau - c_1(1-\delta)\tau})$.

$$N_2 = \frac{b}{A}e^{(c_1-c_2)(1-\delta)\tau}, \quad \Lambda = e^{k_1\delta\tau - c_2(1-\delta)\tau}$$

are two asymptotes, see Fig 2. Λ is strictly decreasing with respect to N_2. For the second function in (2.6), Λ is strictly increasing with respect to N_2. $P_3(-A/a,0)$ and $P_4(0,e^{k_2\delta\tau - c_1(1-\delta)\tau}/h)$ are two fixed points on the solution curve,

$$N_2 = -\frac{A}{a}e^{(c_1-c_2)(1-\delta)\tau}, \quad \Lambda = \frac{1}{h}e^{k_2\delta\tau - c_2(1-\delta)\tau}$$

are two asymptotes. So in the following, we will simulate the solution curve (N_2,Λ) in (2.6) based on different positions of P_2 and P_4.

Therefore, it follows from mathematical analysis and the image method that the principal eigenvalue λ of problem (2.2) exists with a positive eigenfunction pair (ϕ,ψ).

For the scalar case, the corresponding principal eigenvalue can be explicitly expressed.

Example 2.1. Consider the following logistic-type seasonal succession problem with harvesting pulse:

$$\begin{cases} u_t - du_{xx} = au - bu^2, & t\in\Omega^r_{0,\rho}, \quad x\in(-L,L), \\ u_t = -ku, & t\in\Omega^l_0, \quad x\in(-L,L), \\ u\big(((1-\delta)\rho\tau)^+,x\big) = H\big(u((1-\delta)\rho\tau,x)\big), & x\in(-L,L), \\ u(t,-L) = u(t,L) = 0, & t\in(0,+\infty), \\ u(0,x) = u_0(x), & x\in[-L,L], \quad n=0,1,2,\dots, \end{cases} \tag{2.7}$$

where the harvesting function $H(u)$ meets the requirement of [11]. Its corresponding principal eigenvalue problem is

$$\begin{cases} \phi_t - d\phi_{xx} = a\phi + \lambda\phi, & t\in\Omega^r_{0,\rho}, \quad x\in(-L,L), \\ \phi_t = -k\phi + \lambda\phi, & t\in\Omega^l_0, \quad x\in(-L,L), \\ \phi\big(((1-\delta)\rho\tau)^+,x\big) = H'(0)\phi\big((1-\delta)\rho\tau,x\big), & x\in(-L,L), \\ \phi(t,-L) = \phi(t,L) = 0, & t\in[0,\tau], \\ \phi(0,x) = \phi(\tau,x), & x\in[-L,L]. \end{cases} \tag{2.8}$$

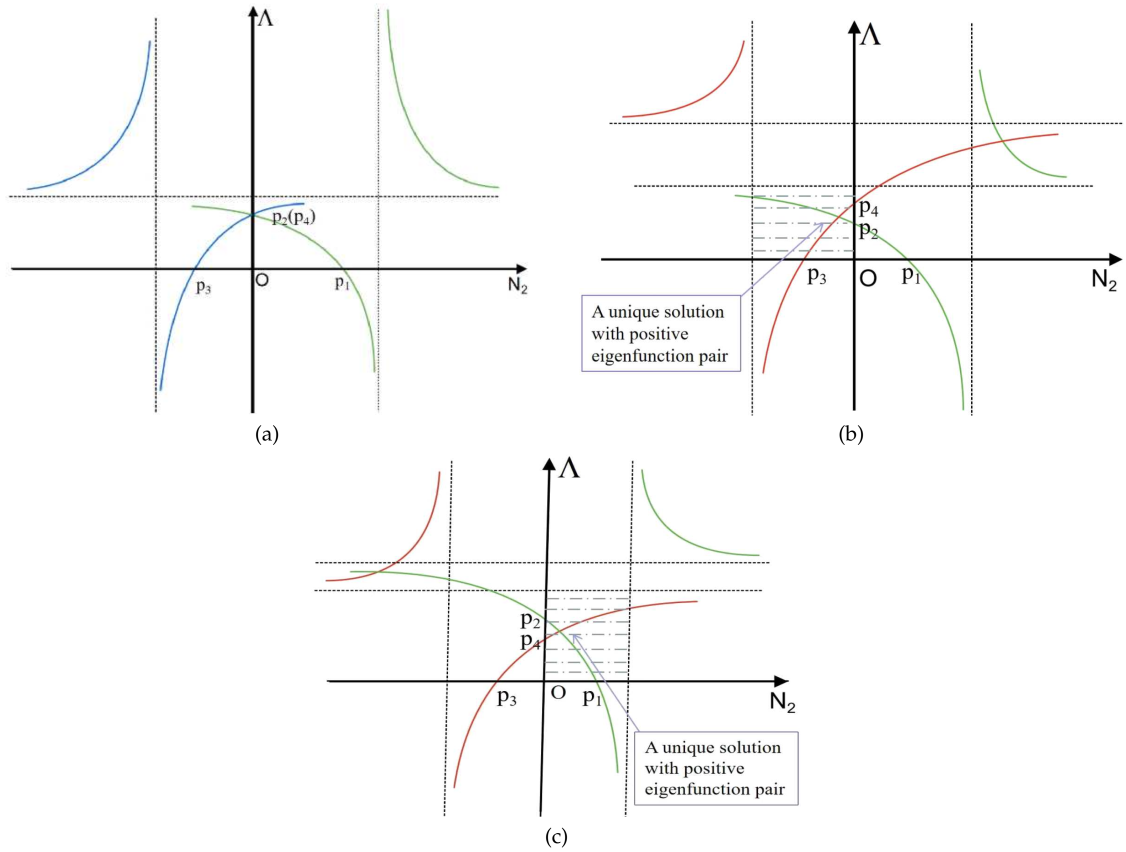


Figure 2: The solution curve of (N_2,Λ) in (2.6) in different cases. Graph (a) is the case of $k_2=(1/(\delta\tau))\ln h+k_1$ (points P_2 and P_4 coincide), graph (b) is the case of $k_2>(1/(\delta\tau))\ln h+k_1$ (point P_4 is above point P_2) and graph (c) is the case of $k_2<(1/(\delta\tau))\ln h+k_1$ (point P_4 is below point P_2), respectively.

The principal eigenvalue can be precisely expressed as

$$\lambda=k\delta-\left(a-d\frac{\pi^2}{4L^2}\right)(1-\delta)-\frac{\ln H'(0)}{\tau}.$$

Proof. We can transform (2.8) into the following spatially independent eigenvalue problem:

$$\begin{cases} f'(t)+d\lambda_0 f=af+\lambda f, & t\in\Omega^r_{0,\rho}, & \text{(2.9a)}\\ f'(t)=-kf+\lambda f, & t\in\Omega^l_0, & \text{(2.9b)}\\ f\big(((1-\delta)\rho\tau)^+\big)=H'(0)f\big((1-\delta)\rho\tau\big), & & \text{(2.9c)}\\ f(0)=f(\tau), & & \text{(2.9d)}\end{cases}$$

where λ_0 is the principal eigenvalue of $-\Delta$ in the time independent problem with Dirichlet boundary condition, and can be computed to $\lambda_0=(\pi/(2L))^2$.

Direct calculations for the Eq. (2.9b) yield

$$f(t)=f\big((1-\delta)\tau\big)e^{(-k+\lambda)(t-(1-\delta)\tau)},\quad t\in\Omega_0^l,$$

hence

$$f\big((1-\delta)\tau\big)=f(\tau)e^{(k-\lambda)\delta\tau}. \tag{2.10}$$

Next, integrating both sides of the Eq. (2.9a) over $t\in\Omega_{0,\rho}^r$ yields

$$\ln\frac{f\big((1-\delta)\rho\tau\big)}{f(0)}+\ln\frac{f\big((1-\delta)\tau\big)}{f\big(((1-\delta)\rho\tau)^+\big)}=(a+\lambda-d\lambda_0)(1-\delta)\tau,$$

which together with the Eqs. (2.9c), (2.9d) and (2.10) results in

$$\lambda=k\delta-\left(a-d\frac{\pi^2}{4L^2}\right)(1-\delta)-\frac{\ln H'(0)}{\tau}.$$

To stress the impact of the coefficients, we denote $\lambda(\delta;H'(0);\tau):=\lambda$. One easily checks that $\lambda(\delta;H'(0);\tau)$ is strictly increasing in δ (the duration of the bad season) if $a>d\pi^2/(4L^2)$, also, $\lambda(\delta;H'(0);\tau)$ is strictly increasing in $1-H'(0)$ (harvesting intensity) and strictly decreasing in τ (period of harvesting). In particular, if $\delta=0$, then

$$\lambda=d\frac{\pi^2}{4L^2}-a-\frac{\ln H'(0)}{\tau},$$

which is investigated in [15]. If $H'(0)=1$, then

$$\lambda=\left(d\frac{\pi^2}{4L^2}-a\right)(1-\delta)+k\delta,$$

which is discussed in [17]. □

In the following, we investigate the properties of the principal eigenvalue λ in (2.2). To emphasize the impact of the coefficients, we denote $\lambda:=\lambda(\delta,h,(-L,L))$.

Let us first introduce the auxiliary problem

$$\begin{cases}
-\phi_t^*-d_1\phi_{xx}^*=b\psi^*-(a+m_1)\phi^*+\lambda^O\phi^*, & t\in\Omega_{0,\rho}^r,\quad x\in(-L,L), & \text{(2.11a)}\\
-\psi_t^*-d_2\psi_{xx}^*=a\phi^*-m_2\psi^*+\lambda^O\psi^*, & t\in\Omega_{0,\rho}^r,\quad x\in(-L,L), & \text{(2.11b)}\\
-\phi_t^*=-k_1\phi^*+\lambda^O\phi^*, & t\in\Omega_0^l,\quad x\in(-L,L), & \text{(2.11c)}\\
-\psi_t^*=-k_2\psi^*+\lambda^O\psi^*, & t\in\Omega_0^l,\quad x\in(-L,L), & \text{(2.11d)}\\
\quad\phi^*\big(((1-\delta)\rho\tau)^+,x\big)=\psi^*\big((1-\delta)\rho\tau,x\big), & x\in(-L,L), & \text{(2.11e)}\\
\quad\psi^*\big(((1-\delta)\rho\tau)^+,x\big)=1/h\psi^*\big((1-\delta)\rho\tau,x\big), & x\in(-L,L), & \text{(2.11f)}\\
\quad\phi^*(t,\pm L)=\psi^*(t,\pm L)=0, & t\in[0,\tau], & \text{(2.11g)}\\
\quad\phi^*(0,x)=\phi^*(\tau,x),\quad \psi^*(0,x)=\psi^*(\tau,x), & x\in[-L,L]. & \text{(2.11h)}
\end{cases}$$

Theorem 2.1. *Problems* (2.2) *and* (2.11) *have the same principal eigenvalue, that is,* $\lambda=\lambda^O$.

Proof. Multiplying the Eq. (2.2a) by ϕ^* and the Eq. (2.11a) by ϕ yield

$$\begin{cases} \phi_t\phi^*-d_1\phi_{xx}\phi^*=b\psi\phi^*-(a+m_1)\phi\phi^*+\lambda\phi\phi^*, \\ -\phi_t^*\phi-d_1\phi_{xx}^*\phi=b\psi^*\phi-(a+m_1)\phi\phi^*+\lambda^O\phi^*\phi. \end{cases}$$

Subtracting these two equations and integrating both sides over $t\in(0,(1-\delta)\tau)$ and $x\in(-L,L)$ result in

$$\begin{aligned}&\int_{-L}^{L}(\phi\phi^*)\big|_{t=0}^{t=(1-\delta)\tau}dx\\ &=b\int_0^{(1-\delta)\tau}\int_{-L}^{L}(\psi\phi^*-\psi^*\phi)dxdt+(\lambda-\lambda^O)\int_0^{(1-\delta)\tau}\int_{-L}^{L}\phi\phi^*dxdt. \end{aligned}\tag{2.12}$$

For the Eqs. (2.2c) and (2.11c), the same procedure and integrating over $t\in((1-\delta)\tau,\tau]$ and $x\in(-L,L)$ yields

$$\int_{-L}^{L}(\phi\phi^*)\big|_{t=(1-\delta)\tau}^{t=\tau}dx=(\lambda-\lambda^O)\int_{(1-\delta)\tau}^{\tau}\int_{-L}^{L}\phi\phi^*dxdt. \tag{2.13}$$

Adding (2.12) and (2.13) now yields

$$0=\int_0^{(1-\delta)\tau}\int_{-L}^{L}(\psi\phi^*-\psi^*\phi)dxdt+(\lambda-\lambda^O)\int_0^{\tau}\int_{-L}^{L}\frac{\phi\phi^*}{b}dxdt. \tag{2.14}$$

Similarly, for the Eqs. (2.2b) and (2.11b), the Eqs. (2.2d) and (2.11d), which together with the impulsive and periodic conditions in (2.2) and (2.11), we have that

$$\begin{aligned}0=&\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^{(1-\delta)\tau}\right)\int_{-L}^{L}(\phi\psi^*-\phi^*\psi)dxdt\\ &+(\lambda-\lambda^O)\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^{\tau}\right)\int_{-L}^{L}\frac{\psi\psi^*}{a}dxdt. \end{aligned}\tag{2.15}$$

Finally, adding the Eqs. (2.14) and (2.15) results in

$$0=(\lambda-\lambda^O)\left[\int_0^{\tau}\int_{-L}^{L}\frac{\phi\phi^*}{b}dxdt+\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^{\tau}\right)\int_{-L}^{L}\frac{\psi\psi^*}{a}dxdt\right],$$

which means that $\lambda=\lambda^O$ since that $\phi\phi^*$ and $\psi\psi^*$ are positive. □

In the following, the properties of $\lambda:=\lambda(\delta,h,(-L,L))$ can be derived by the eigenvalue problem (2.2) and the auxiliary problem (2.11).

Theorem 2.2. *The following statements hold:*

(i) $\lambda(\delta,h,(-L,L))$ *is strictly monotonic decreasing with respect to* L *for any given* δ *and* h.

(ii) $\lambda(\delta,h,(-L,L))$ *is strictly monotonic decreasing with respect to* h *for any given* L *and* δ.

(iii) $\lambda(\delta,h,(-L,L))$ *is strictly monotonic increasing with respect to* δ *provided that* $k_1 > a+m_1+d_1\lambda_0$ *and* $k_2>m_2+d_2\lambda_0$ *for any given* L *and* h.

Proof. We first prove (i). Suppose that $0<L_1\le L_2$. For simplicity, we denote $\lambda(L_i):=\lambda(\delta,h,(-L_i,L_i)),i=1,2$. Let $(\phi,\psi,\lambda(L_2))$ and $(\phi^*,\psi^*,\lambda(L_1))$ denote the principal eigenpair to problem (2.2) in $[-L_2,L_2]$, respectively (2.11) in $[-L_1,L_1]$.

As in the proof of Theorem 2.1, by the method of "Multiplying-Multiplying-Subtracting-Integrating" for the equations involving ϕ,ϕ^* over $(0,(1-\delta)\tau]\times[-L_1,L_1]$ and $(t,x)\in((1-\delta)\tau,\tau]\times[-L_1,L_1]$, respectively, and then adding these two equations, (2.14) can be written as

$$
\begin{aligned}
& d_1\int_0^{(1-\delta)\tau}\left(\frac{\phi_x^*\phi}{b}\right)\Big|_{x=-L_1}^{x=L_1}dt \\
=&\int_0^{(1-\delta)\tau}\int_{-L_1}^{L_1}(\psi\phi^*-\psi^*\phi)dxdt+\big(\lambda(L_2)-\lambda(L_1)\big)\int_0^{\tau}\int_{-L_1}^{L_1}\frac{\phi\phi^*}{b}dxdt.
\end{aligned}
$$

Similarly, (2.15) can be written as

$$
\begin{aligned}
& d_2\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^{(1-\delta)\tau}\right)\left(\frac{\psi_x^*\psi}{a}\right)\Big|_{x=-L_1}^{x=L_1}dt \\
=&\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^{(1-\delta)\tau}\right)\int_{-L_1}^{L_1}(\phi\psi^*-\phi^*\psi)dxdt \\
&+\big(\lambda(L_2)-\lambda(L_1)\big)\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^{\tau}\right)\int_{-L_1}^{L_1}\frac{\psi\psi^*}{a}dxdt.
\end{aligned}
$$

Adding these two equations yields

$$
\lambda(L_2)-\lambda(L_1)=\frac{d_1\int_0^{(1-\delta)\tau}\left(\frac{\phi_x^*\phi}{b}\right)\Big|_{x=-L_1}^{x=L_1}dt+d_2\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^{(1-\delta)\tau}\right)\left(\frac{\psi_x^*\psi}{a}\right)\Big|_{x=-L_1}^{x=L_1}dt}{\int_0^{\tau}\int_{-L_1}^{L_1}\frac{\phi\phi^*}{b}dxdt+\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^{\tau}\right)\int_{-L_1}^{L_1}\frac{\psi\psi^*}{a}dxdt},
$$

so $\lambda(L_2)-\lambda(L_1)<0$ since $\phi_x^*|_{x=-L_1}^{x=L_1}<0$ and $\psi_x^*|_{x=-L_1}^{x=L_1}<0$, by the strong maximum principle.

We next prove (ii). Regarding h as a variable, then differentiating both sides of the equations in problem (2.2) with respect to h, yields

$$\begin{cases}\phi_t'-d_1\phi_{xx}'=b\psi'-(a+m_1)\phi'+\lambda'\phi+\lambda\phi', & t\in\Omega_{0,\rho}^r,\quad x\in(-L,L), \quad (2.16a)\\ \psi_t'-d_2\psi_{xx}'=a\phi'-m_2\psi'+\lambda'\psi+\lambda\psi', & t\in\Omega_{0,\rho}^r,\quad x\in(-L,L), \quad (2.16b)\\ \phi_t'=-k_1\phi'+\lambda'\phi+\lambda\phi', & t\in\Omega_0^l,\quad x\in(-L,L), \quad (2.16c)\\ \psi_t'=-k_2\psi'+\lambda'\psi+\lambda\psi', & t\in\Omega_0^l,\quad x\in(-L,L), \quad (2.16d)\\ \phi'\big(((1-\delta)\rho\tau)^+,x\big)=\phi\big((1-\delta)\rho\tau,x\big), & x\in(-L,L), \quad (2.16e)\\ \psi'\big(((1-\delta)\rho\tau)^+,x\big)=\psi\big((1-\delta)\rho\tau,x)+h\psi'((1-\delta)\rho\tau,x), & x\in(-L,L), \quad (2.16f)\\ \phi'(t,\pm L)=\psi'(t,\pm L)=0,\quad t\in[0,\tau], & (2.16g)\\ \phi'(0,x)=\phi'(\tau,x),\psi'(0,x)=\psi'(\tau,x), & x\in[-L,L]. \quad (2.16h)\end{cases}$$

Multiplying the Eq. (2.16a) (Eq. (2.16c)) by ϕ^* and the Eq. (2.11a) (Eq. (2.11c)) by ϕ', then subtracting and integrating over $(0,(1-\delta)\tau]\times[-L,L]$ $(((1-\delta)\tau,\tau]\times[-L,L])$, respectively, and finally adding them results in

$$0=\int_0^{(1-\delta)\tau}\int_{-L}^{L}(\psi'\phi^*-\psi^*\phi')dxdt+\lambda'\int_0^{\tau}\int_{-L}^{L}\frac{\phi\phi^*}{b}dxdt.$$

Similarly, for the equations involving ψ' and ψ^* in (2.16) and (2.11), we have

$$\begin{aligned}&\int_{-L}^{L}\frac{-1}{ah}\psi\big((1-\delta)\rho\tau\big)\psi^*\big((1-\delta)\rho\tau\big)dx\\ =&\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^{(1-\delta)\tau}\right)\int_{-L}^{L}(\phi'\psi^*-\phi^*\psi')dxdt\\ &+\lambda'\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^{\tau}\right)\int_{-L}^{L}\frac{\psi\psi^*}{a}dxdt.\end{aligned}$$

Thus, adding these two equations we obtain

$$\lambda'=\frac{\dfrac{-1}{ah}\displaystyle\int_{-L}^{L}\psi\big((1-\delta)\rho\tau\big)\psi^*\big((1-\delta)\rho\tau\big)dx}{\displaystyle\int_0^{\tau}\int_{-L}^{L}\frac{\phi\phi^*}{b}dxdt+\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^{\tau}\right)\int_{-L}^{L}\frac{\psi\psi^*}{a}dxdt}<0.$$

We finally prove (iii). Suppose that $0<\delta_1<\delta_2<1$. For simplicity, we denote $\lambda(\delta_i):=\lambda(\delta_i,h,(-L,L)),i=1,2$.

Recalling the method of separation, we first consider the following auxiliary problems:

$$\begin{cases} f_{1t}=bf_2-(a+m_1+d_1\lambda_0)f_1+\lambda(\delta_1)f_1, & t\in\Omega^r_{0,\rho}, & (2.17a)\\ f_{2t}=af_1-(m_2+d_2\lambda_0)f_2+\lambda(\delta_1)f_2, & t\in\Omega^r_{0,\rho}, & (2.17b)\\ f_{1t}=-k_1f_1+\lambda(\delta_1)f_1, & t\in\big((1-\delta_1)\tau,\tau\big], & (2.17c)\\ f_{2t}=-k_2f_2+\lambda(\delta_1)f_2, & t\in\big((1-\delta_1)\tau,\tau\big], & (2.17d)\\ f_1\big(((1-\delta_1)\rho\tau)^+\big)=f_1\big((1-\delta_1)\rho\tau\big), & & (2.17e)\\ f_2\big(((1-\delta_1)\rho\tau)^+\big)=hf_2\big((1-\delta_1)\rho\tau\big), & & (2.17f)\\ f_1(0)=f_1(\tau),\quad f_2(0)=f_2(\tau), & & (2.17g)\end{cases}$$

and

$$\begin{cases} -f^*_{1t}=bf^*_2-(a+m_1+d_1\lambda_0)f^*_1+\lambda(\delta_2)f^*_1, & t\in\Omega^r_{0,\rho}, & (2.18a)\\ -f^*_{2t}=af^*_1-(m_2+d_2\lambda_0)f^*_2+\lambda(\delta_1)f^*_2, & t\in\Omega^r_{0,\rho}, & (2.18b)\\ -f^*_{1t}=-k_1f^*_1+\lambda(\delta_1)f^*_1, & t\in\big((1-\delta_2)\tau,\tau\big], & (2.18c)\\ -f^*_{2t}=-k_2f^*_2+\lambda(\delta_1)f^*_2, & t\in\big((1-\delta_2)\tau,\tau\big], & (2.18d)\\ f^*_1\big(((1-\delta_1)\rho\tau)^+\big)=f^*_1\big((1-\delta_2)\rho\tau\big), & & (2.18e)\\ f^*_2\big(((1-\delta_1)\rho\tau)^+\big)=\dfrac{1}{h}f^*_2\big((1-\delta_2)\rho\tau\big), & & (2.18f)\\ f^*_1(0)=f^*_1(\tau),\quad f^*_2(0)=f^*_2(\tau) & & (2.18g)\end{cases}$$

corresponding to (2.2) and (2.11), respectively.

Multiplying the Eq. (2.18a) by f_1, then integrating over $t\in(0,(1-\delta_2)\tau]$ gives

$$\begin{aligned}&-f^*_1f_1\big|_{t=0}^{t=(1-\delta_2)\tau}+\int_0^{(1-\delta_2)\tau}f^*_1f_{1t}dt\\ =&\int_0^{(1-\delta_2)\tau}\big(bf^*_2f_1-(a+m_1+d_1\lambda_0)f^*_1f_1\big)dt+\lambda(\delta_2)\int_0^{(1-\delta_2)\tau}f^*_1f_1dt.\end{aligned}$$

Since

$$f_{1t}=bf_2-(a+m_1+d_1\lambda_0)f_1+\lambda(\delta_1)f_1,\quad t\in\big(0,(1-\delta_2)\tau\big],$$

so

$$-f^*_1f_1\big|_{t=0}^{t=(1-\delta_2)\tau}=b\int_0^{(1-\delta_2)\tau}(f^*_2f_1-f^*_1f_2)dt+\big(\lambda(\delta_2)-\lambda(\delta_1)\big)\int_0^{(1-\delta_2)\tau}f^*_1f_1dt.$$

Similarly, multiplying the Eq. (2.18c) with f_1 and integrating over $t\in((1-\delta_2)\tau,\tau]$, by considering the Eq. (2.17c), one easily checks that

$$
\begin{aligned}
&-f_1^*f_1\big|_{t=(1-\delta_2)\tau}^{t=\tau}+\int_{(1-\delta_2)\tau}^{(1-\delta_1)\tau} f_1^*[bf_2-(a+m-1+d_1\lambda_0)f_1]dt\\
=&k_1\int_{(1-\delta_1)\tau}^{(1-\delta_2)\tau} f_1^*f_1+\big(\lambda(\delta_2)-\lambda(\delta_1)\big)\int_{(1-\delta_2)\tau}^{\tau} f_1^*f_1dt.
\end{aligned}
$$

Adding the two equations above, we find that

$$
\begin{aligned}
&\big(\lambda(\delta_2)-\lambda(\delta_1)\big)\int_0^\tau \frac{f_1^*f_1}{b}dt\\
=&\int_{(1-\delta_2)\tau}^{(1-\delta_1)\tau} f_1^*\left[f_2-\frac{(a+m_1+d_1\lambda_0)f_1}{b}\right]dt\\
&-\int_0^{(1-\delta_2)\tau}(f_2^*f_1-f_1^*f_2)+k_1\int_{(1-\delta_2)\tau}^{(1-\delta_1)\tau}\frac{f_1^*f_1}{b}dt.
\end{aligned}
$$

By the same procedure, for the Eqs. (2.18b) and (2.18d), we can carefully calculate that

$$
\begin{aligned}
&\big(\lambda(\delta_2)-\lambda(\delta_1)\big)\left(\int_0^{(1-\delta_2)\rho\tau}+\int_{((1-\delta_2)\rho\tau)^+}^{\tau}\right)\frac{f_2^*f_2}{a}dt\\
=&\int_{(1-\delta_2)\tau}^{(1-\delta_1)\tau} f_2^*\left[f_1-\frac{(m_2+d_2\lambda_0)f_2}{a}\right]dt\\
&-\left(\int_0^{(1-\delta_2)\rho\tau}+\int_{((1-\delta_2)\rho\tau)^+}^{(1-\delta_2)\tau}\right)(f_1^*f_2-f_2^*f_1)dt+k_2\int_{(1-\delta_2)\tau}^{(1-\delta_1)\tau}\frac{f_2^*f_2}{a}dt.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&\big(\lambda(\delta_2)-\lambda(\delta_1)\big)\left[\int_0^\tau\frac{f_1^*f_1}{b}dt+\left(\int_0^{(1-\delta_2)\rho\tau}+\int_{((1-\delta_2)\rho\tau)^+}^{\tau}\right)\frac{f_2^*f_2}{a}dt\right]\\
>&\int_{(1-\delta_2)\tau}^{(1-\delta_1)\tau}\frac{f_1^*f_1}{b}[k_1-(a+m_1+d_1\lambda_0)]dt+\int_{(1-\delta_2)\tau}^{(1-\delta_1)\tau}\frac{f_2^*f_2}{a}[k_2-(m_2+d_2\lambda_0)]dt>0
\end{aligned}
$$

provided that $k_1>a+m_1+d_1\lambda_0$ and $k_2>m_2+d_2\lambda_0$. □

Based on the principal eigenvalue of problem (2.2), we have the following results for problem (2.1).

Theorem 2.3. *The following statements are true:*

(i) If $\lambda(\delta,h,(-L,L))\geq 0$, then the periodic problem (2.1) *has no nonnegative nontrivial solution.*

(ii) If $\lambda(\delta,h,(-L,L))<0$, then the periodic problem (2.1) *admits a unique positive solution.*

Proof. (i) If $\lambda(\delta,h,(-L,L))\geq 0$, and we assume that the periodic problem (2.1) has a nonnegative nontrivial solution (U,V), then the solution is positive by strong maximum principle. As in the proof of Theorem 2.2, we can use the method of "Multiplying-Multiplying-Subtracting-Integrating" to derive that

$$\begin{aligned}0=&-\lambda\left[\int_0^\tau\int_{-L}^L\frac{U\phi^*}{b}dxdt+\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^\tau\right)\int_{-L}^L\frac{V\psi^*}{a}dxdt\right]\\&-\alpha_1\int_0^\tau\int_{-L}^L\frac{U^2\phi^*}{b}dxdt-\alpha_2\left(\int_0^{(1-\delta)\rho\tau}+\int_{((1-\delta)\rho\tau)^+}^\tau\right)\int_{-L}^L\frac{V^2\psi^*}{a}dxdt,\end{aligned}$$

which leads to a contradiction since U,V,ϕ^* and ψ^* are positive.

If $\lambda(\delta,h,(-L,L))<0$, we split the proof into two steps to prove the existence and uniqueness of solution to problem (2.1).

Step 1. A pair of upper and lower solutions to the problem (2.1) is constructed. Let

$$\overline{U}(t,x)=\overline{V}(t,x)=C^*\geq\max\left\{\left|\frac{-a-m_1+b}{\alpha_1}\right|,\left|\frac{-m_2+a}{\alpha_2}\right|\right\}. \tag{2.19}$$

It is obvious that for $t\in\Omega^r_{0,\rho}$ and $x\in(-L,L)$, we have

$$\begin{aligned}&\overline{U}_t-d_1\overline{U}_{XX}-b\overline{V}+(a+m_1)\overline{U}+\alpha_1\overline{U}^2\\&=C^*(\alpha_1C^*+a+m_1+\rho_m-b)\geq 0,\\&\overline{V}_t-d_2\overline{V}_{xx}-a\overline{U}+m_2\overline{V}+\alpha_2\overline{V}^2\\&=-hC^*+\left(m_2+\frac{N\dot{\rho}(t)}{\rho(t)}\right)C^*+\alpha_2C^{*2}\\&\geq C^*(\alpha_2C^*+m_2-a)\geq 0.\end{aligned}$$

Also, one easily checks that for $t\in\Omega^l_0$ and $x\in(-L,L)$,

$$\overline{U}_t+k_1\overline{U}=k_1C^*>0,\quad \overline{V}_t+k_2\overline{V}=k_2C^*>0$$

hold. Furthermore, the boundary condition $\overline{U}(t,\pm L)=\overline{V}(t,\pm L)>0$, the pulse condition for juveniles

$$\overline{U}\big(((1-\delta)\rho\tau)^+,x\big)-\overline{U}\big((1-\delta)\rho\tau,x\big)=0$$

as well as for adults

$$\overline{V}\big(((1-\delta)\rho\tau)^+,x\big)-h\overline{V}\big((1-\delta)\rho\tau,x\big)=C^*(1-h)\geq 0$$

hold due to $0<h\leq 1$, so $(\overline{U}(t,x),\overline{V}(t,x))$ is an upper solution to problem (2.1) by the comparison principle.

In what follows, let us consider a lower solution. Define

$$\underline{U}(t,x)=\varepsilon\phi(t,x),\quad \underline{V}(t,x)=\varepsilon\psi(t,x)$$

for $[0,\tau]\times[-L,L]$, where (ϕ,ψ) is defined in (2.2) and $\varepsilon(>0)$ is sufficiently small and to be chosen later.

Direct calculations yield

$$\begin{aligned}&\underline{U}_t-d_1\underline{U}_{XX}-b\underline{V}+(a+m_1)\underline{U}+\alpha_1\underline{U}^2\\&=\varepsilon\big(\phi_t-d_phi_{xx}-b\psi+(a+m_1)\phi+\alpha_1\varepsilon\phi^2\big)\\&=\varepsilon\phi(\lambda+\alpha_1\varepsilon\phi)\le 0\end{aligned}$$

for $t\in\Omega^r_{0,\rho}$ and $x\in(-L,L)$ provided that $\varepsilon\le-\lambda/(\alpha_1\|\phi\|)$. Similarly,

$$\underline{V}_t-d_2\underline{V}_{xx}-a\underline{U}+m_2\underline{V}+\alpha_2\underline{V}^2\le 0$$

provided that $\varepsilon\le-\lambda/(\alpha_2\|\psi\|)$. So we here choose

$$\varepsilon:=\min\left\{\frac{-\lambda}{\alpha_1\|\phi\|},\frac{-\lambda}{\alpha_2\|\psi\|}\right\}.$$

Now, since $\lambda<0$, for $t\in\Omega^l_0$ and $x\in(-L,L)$, we obtain

$$\underline{U}_t+k_1\underline{U}=\varepsilon\lambda\phi<0,\quad \underline{U}_t+k_1\underline{U}=\varepsilon\lambda\psi<0.$$

Moreover,

$$\begin{aligned}&\underline{V}\big(((1-\delta)\rho\tau)^+,x\big)-h\underline{V}\big((1-\delta)\rho\tau,x\big)\\&=\varepsilon\big[\psi\big(((1-\delta)\rho\tau)^+,x\big)-h\psi\big((1-\delta)\rho\tau,x\big)\big]\\&=\varepsilon\big[h\psi\big((1-\delta)\rho\tau,x\big)-h\psi\big((1-\delta)\rho\tau,x\big)\big]=0\end{aligned}$$

holds for $x\in(-L,L)$. Recalling (2.2), we have

$$\underline{U}(t,\pm L)=\underline{V}(t,\pm L)=0,\quad t\in[0,\tau],$$

and

$$\underline{U}(0,x)=\underline{U}(\tau,x),\quad \underline{V}(0,x)=\underline{V}(\tau,x),\quad x\in[-L,L].$$

It finally follows from the comparison principle that $(\underline{U},\underline{V})$ is a lower solution to problem (2.1).

Step 2. We prove the existence and uniqueness of positive periodic solutions.

For simplicity, we first denote

$$\begin{aligned}&f_1(t,U,V):=bV-(a+m_1)U-\alpha_1U^2,\\&f_2(t,U,V):=aU-m_2V-\alpha_2V^2,\end{aligned}\tag{2.20}$$

then there exists C_1 and C_2 large enough such that

$$\begin{aligned}&F_1\triangleq f_1(t,U,V)+C_1U, F_3\triangleq -k_1U+C_1U,\\&F_2\triangleq f_2(t,U,V)+C_2V, F_4\triangleq -k_2V+C_2V\end{aligned}$$

are increasing with respect to U and V, respectively.

In what follows, the pair $(\overline{U}^{(0)},\overline{V}^{(0)}):=(\overline{U},\overline{V})$ is used to construct the iteration sequence $\{\overline{U}^{(n)},\overline{V}^{(n)}\}$, which satisfies

$$\begin{cases}\overline{U}_t^{(n)}-d_1\overline{U}_{xx}^{(n)}+C_1\overline{U}^{(n)}=F_1\big(t,\overline{U}^{(n-1)},\overline{V}^{(n-1)}\big), & t\in\Omega_{0,\rho}^r, \quad x\in(-L,L),\\ \overline{V}_t^{(n)}-d_2\overline{V}^{(n)}+C_2\overline{V}^{(n)}=F_2\big(t,\overline{U}^{(n-1)},\overline{V}^{(n-1)}\big), & t\in\Omega_{0,\rho}^r, \quad x\in(-L,L),\\ \overline{U}_t^{(n)}+C_1\overline{U}^{(n)}=F_3\big(t,\overline{U}^{(n-1)},\overline{V}^{(n-1)}\big), & t\in\Omega_0^l, \quad x\in(-L,L),\\ \overline{U}_t^{(n)}+C_1\overline{U}^{(n)}=F_3\big(t,\overline{U}^{(n-1)},\overline{V}^{(n-1)}\big), & t\in\Omega_0^l, \quad x\in(-L,L),\\ \overline{U}^{(n)}(t,\pm L)=\overline{V}^{(n)}(t,\pm L)=0, & t\in[0,\tau]\end{cases} \tag{2.21}$$

with periodic conditions

$$\overline{U}^{(n)}(0,x)=\overline{U}^{(n-1)}(\tau,x), \quad \overline{V}^{(n)}(0,x)=\overline{V}^{(n-1)}(\tau,x), \quad x\in(-L,L), \tag{2.22}$$

and impulsive conditions

$$\begin{cases}\overline{U}^{(n)}\big(((1-\delta)\rho\tau)^+,x\big)=\overline{U}^{(n-1)}\big((1-\delta)\rho\tau+\tau,x\big), & x\in(-L,L),\\ \overline{V}^{(n)}\big(((1-\delta)\rho\tau)^+,x\big)=h\overline{V}^{(n-1)}\big((1-\delta)\rho\tau+\tau,x\big), & x\in(-L,L).\end{cases} \tag{2.23}$$

Similarly, $\{\underline{U}^{(n)},\underline{V}^{(n)}\}$ can be constructed with $\{\underline{U}^{(0)},\underline{V}^{(0)}\}:=(\underline{U},\underline{V})$. Then for $t\geq 0$ and $x\in(-l,L)$, we derive from [16, Theorem 2.1] that

$$\begin{aligned}(\underline{U},\underline{V})&\leq(\underline{U}^{(n-1)},\underline{V}^{(n-1)})\leq(\underline{U}^{(n)},\underline{V}^{(n)})\\ &\leq(\overline{U}^{(n)},\overline{V}^{(n)})\leq(\overline{U}^{(n-1)},\overline{V}^{(n-1)})\leq(\overline{U},\overline{V}).\end{aligned}$$

Therefore, it is easy to see that $(\overline{U}^{(n)},\overline{V}^{(n)})$ and $(\underline{U}^{(n)},\underline{V}^{(n)})$ converge to the periodic solution of problem (2.1) as $n\to\infty$, which are denoted by $(\overline{U}^*,\overline{V}^*)$ and $(\underline{U}^*,\underline{V}^*)$, respectively. Moreover, we claim that $(\underline{U}^*,\underline{V}^*)$ and $(\overline{U}^*,\overline{V}^*)$ are minimal and maximal periodic solutions to the problem (2.1), respectively. The uniqueness of the solutions to problem (2.1), say (U^*,V^*), can be verified by a proof of contradiction as in [25]. □

3 Asymptotic behavior of the solution

In order to investigate the longtime behavior of solution to problem (1.1), we first prove the existence and uniqueness of the global solution.

Theorem 3.1. *For any given nonnegative initial value $u_{i,0}(x)\in C^2([-L,L])$ satisfying $u_{i,0}(\pm L)=0$ with $i=1,2$, problem* (1.1) *admits a unique solution (u_1,u_2) defined for all $t\in(0,+\infty)$. Moreover,*

$$u_1(t,x),\ u_2(t,x)\in C^{1,2}\big((\Omega_{n,\rho}^r\textstyle\bigcup\Omega_n^l)\times[-L,L]\big), \quad n=0,1,2,\ldots.$$

Proof. (i) For the interval $t \in (0,(1-\delta)\rho\tau]$, since $u_{i,0}(x) \in C^2([-L,L])$, recalling the existence and uniqueness of the solution without pulse in [14] and Lemma 3.1 later, we see that the solution (u_1,u_2) to problem (1.1) exists and is unique for $t \in (0^+,(1-\delta)\rho\tau]$. Furthermore, $u_i \in C^{1,2}((0,(1-\delta)\rho\tau]\times[-L,L])$ for $i=1,2$.

(ii) For the interval $t\in((1-\delta)\rho\tau,(1-\delta)\tau]$, we regard

$$\Big(u_1\big(((1-\delta)\rho\tau)^+,x\big),u_2\big(((1-\delta)\rho\tau)^+,x\big)\Big)$$

as the initial value of the solution $(u_1(t,x),u_2(t,x))$ to problem (1.1). Since $u_i((1-\delta)\rho\tau,x)\in C^2([-L,L])$, we derive that the new initial value satisfies that

$$\begin{aligned} &u_1\big(((1-\delta)\rho\tau)^+,x\big)=u_1\big((1-\delta)\rho\tau,x\big)\in C^2([-L,L]) \\ &u_2\big(((1-\delta)\rho\tau)^+,x\big)=hu_2\big((1-\delta)\rho\tau,x\big)\in C^2([-L,L]). \end{aligned}$$

Recalling the existence and uniqueness of the solution without pulse in [14] and Lemma 3.1 later, we see that the solution (u_1,u_2) to problem (1.1) exists and is unique in $t\in(((1-\delta)\rho\tau)^+,(1-\delta)\tau]$. Furthermore,

$$u_i\in C^{1,2}\big(\big((1-\delta)\rho\tau,(1-\delta)\tau\big]\times[-L,L]\big),\quad i=1,2.$$

(iii) For the interval $t\in((1-\delta)\tau,\tau]$, (u_1,u_2) satisfies the ODE system, that is, the Eqs. (1.1c) and (1.1d), since

$$u_i\big((1-\delta)\tau,x\big)\in C^2([-L,L]),$$

and we then have

$$u_i\in C^{1,2}\big(\big((1-\delta)\tau,\tau\big]\times[-L,L]\big),\quad i=1,2.$$

(iv) For the interval $t\in(\tau,2\tau]$, by the same procedure as (i)-(iii), the new initial value satisfies $u_i(\tau,x)\in C^2([-L,L])$. Due to Lemma 3.1, one easily checks that the solution (u_1,u_2) to problem (1.1) exists and is unique in $t\in(\tau,2\tau]$.

(v) The local existence and uniqueness of the solution can be derived by the same process in intervals $t\in(2\tau,3\tau],t\in(3\tau,4\tau],\dots$, and step by step, we then find a maximal time interval $[0,T_{\max})$ with $T_{\max}:=n_0\tau+\tau_0$, $0\le\tau_0<\tau$ and positive integer n_0 by Zorn's lemma such that problem (1.1) admits a unique solution in $[0,T_{\max})$.

(vi) We now claim that $T_{\max}=+\infty$. The following estimates of (u_1,u_2) in Lemma 3.1, which together with the standard continuous extension method, yield the global existence and uniqueness of the solution to problem (1.1). □

Lemma 3.1. *For any given nonnegative integer n_0 and $0<\tau_1\le\tau$, if (u_1,u_2) is a solution to problem* (1.1) *defined for $t\in(0,T]$ with $T:=n_0\tau+\tau_1$, we have that*

$$(0,0)<\big(u_1(t,x),u_2(t,x)\big)\le k\left(\sqrt[3]{\frac{ab^2}{\alpha_1^2\alpha_2}},\sqrt[3]{\frac{a^2b}{\alpha_1\alpha_2^2}}\right)=:k(A_1,A_2)$$

for $t\in(0,\tau]$ and $x\in(-L,L)$, where $k\ge\|u_{1,0}\|_\infty/A_1$ and $k\ge\|u_{2,0}\|_\infty/A_2$.

Proof. Denote $(\bar{u}_1,\bar{u}_2)=k(A_1,A_2)$, careful calculations yield

$$\bar{u}_{1t}-d_1\bar{u}_{1xx}\geq b\bar{u}_2-(a+m_1)\bar{u}_1-\alpha_1\bar{u}_1^2,$$
$$\bar{u}_{2t}-d_2\bar{u}_{2xx}\geq a\bar{u}_1-m_2\bar{u}_2-\alpha_2\bar{u}_2^2$$

for $t\geq 0$ and $x\in(-L,L)$.

We first consider the case that $n_0=0$ and $t\in(0,T]\subseteq(0,\tau]$. Since

$$(u_{1,0},u_{2,0})\leq\big(\bar{u}_1(0,x),\bar{u}_2(0,x)\big),$$

using the comparison principle yields

$$(0,0)<\big(u_1(t,x),u_2(t,x)\big)\leq\big(\bar{u}_1(t,x),\bar{u}_2(t,x)\big)$$

for $t\in(0,\tau_1]$ and $x\in(-L,L)$. It must be mentioned here, if $(1-\delta)\rho\tau<\tau_1\leq(1-\delta)\tau$, the impulsive condition and the assumption that $h\leq 1$ have been used, while if $(1-\delta)\tau<\tau_1\leq\tau$, the comparison principle for the Eqs. (1.1c) and (1.1d) is applied.

For the case $n_0=1$ and $t\in(0,\tau+\tau_1]$. Since the interval $t\in(0,\tau]$ is discussed above, we then have

$$(0,0)<\big(u_1(\tau,x),u_2(\tau,x)\big)\leq(\bar{u}_1,\bar{u}_2)$$

for $x\in(-L,L)$. Let us consider the time interval $t\in(\tau,\tau+\tau_1]$. It follows from the comparison principle that

$$(0,0)<\big(u_1(t,x),u_2(t,x)\big)\leq(\bar{u}_1,\bar{u}_2)$$

for $t\in(\tau,\tau+\tau_1]$ and $x\in(-L,L)$. Step by step,

$$(0,0)<\big(u_1(t,x),u_2(t,x)\big)\leq(\bar{u}_1,\bar{u}_2)$$

for $t\in(0,T]$ and $x\in(-L,L)$ can be obtained. □

Next, some threshold-type results for the global dynamics of model (1.1) are provided.

Theorem 3.2. *The assertions below are valid.*

(i) If $\lambda(\delta,h,(-L,L))\geq 0$, then the solution of model (1.1) *satisfies*

$$\lim_{t\to+\infty}\|u_1(t,\cdot)\|_{C[-L,L]}=\lim_{t\to+\infty}\|u_2(t,\cdot)\|_{C[-L,L]}=0.$$

(ii) If $\lambda(\delta,h,(-L,L))<0$, then the solution of model (1.1) *satisfies*

$$\lim_{m\to+\infty}\big(u_1(t+m\tau,x),u_2(t+m\tau,x)\big)=\big(U^*(t,x),V^*(t,x)\big)$$

uniformly for $(t,x)\in[0,\infty)\times[-L,L]$, where (U^*,V^*) is the unique positive solution of problem (2.1).

Proof. The proof is divided into two steps.

Step 1.
$$\big(\underline{U}^{(0)},\underline{V}^{(0)}\big)(t,x)\le(u_1,u_2)(t,x)\le\big(\overline{U}^{(0)},\overline{V}^{(0)}\big)(t,x)$$
holds for any $t\ge 0$ and $x\in[-L,L]$.

We first assume that $(u_1,u_2)(0,x)>0$ in $[-L,L]$, otherwise we can replace 0 by t_0 for some time $t_0>0$. A sufficiently small ε and a big enough C^* can be chosen such that

$$\big(\underline{U},\underline{V}\big)(0,x)\le(u_1,u_2)(0,x)\le\big(\overline{U},\overline{V}\big)(0,x),\quad x\in[-L,L],$$

which means

$$\big(\underline{U}^{(0)},\underline{V}^{(0)}\big)(0,x)\le(u_1,u_2)(0,x)\le\big(\overline{U}^{(0)},\overline{V}^{(0)}\big)(0,x),\quad x\in[-L,L].$$

Also,

$$\begin{aligned}&\big(\underline{U}^{(0)},\underline{V}^{(0)}\big)\big((1-\delta)\rho\tau,x\big)\\ \le\,&(u_1,u_2)\big((1-\delta)\rho\tau,x\big)\\ \le\,&\big(\overline{U}^{(0)},\overline{V}^{(0)}\big)\big((1-\delta)\rho\tau,x\big),\quad x\in[-L,L],\end{aligned}$$

which together $0<h\le 1$, yields

$$\begin{aligned}&\big(\underline{U}^{(0)},\underline{V}^{(0)}\big)\big(((1-\delta)\rho\tau)^+,x\big)\le(u_1,u_2)\big(((1-\delta)\rho\tau)^+,x\big)\\ \le\,&\big(\overline{U}^{(0)},\overline{V}^{(0)}\big)\big(((1-\delta)\rho\tau)^+,x\big),\quad x\in[-L,L].\end{aligned}$$

So the comparison argument yields that

$$\big(\underline{U}^{(0)},\underline{V}^{(0)}\big)(t,x)\le(u_1,u_2)(t,x)\le\big(\overline{U}^{(0)},\overline{V}^{(0)}\big)(t,x)$$

hold for
$$t\in\big(0,(1-\delta)\rho\tau\big]\cup\big(((1-\delta)\rho\tau)^+,\tau\big],\quad x\in[-L,L].$$
Moreover, we can derive that

$$\big(\underline{U}^{(0)},\underline{V}^{(0)}\big)(t,x)\le(u_1,u_2)(t,x)\le\big(\overline{U}^{(0)},\overline{V}^{(0)}\big)(t,x),\quad t\ge 0,\quad x\in[-L,L].$$

Step 2.
$$\big(\underline{U}^{(1)},\underline{V}^{(1)}\big)(t,x)\le(u_1,u_2)(t+\tau,x)\le\big(\overline{U}^{(1)},\overline{V}^{(1)}\big)(t,x)$$
holds for any $t\ge 0$ and $x\in[-L,L]$.

According to the iteration sequences in Theorem 2.3, it is clear that for $x\in[-L,L]$,

$$\begin{aligned}\big(\underline{U}^{(1)},\underline{V}^{(1)}\big)(0,x)&=\big(\underline{U}^{(0)},\underline{V}^{(0)}\big)(\tau,x)\le(u_1,u_2)(\tau,x)\le\big(\overline{U}^{(0)},\overline{V}^{(0)}\big)(\tau,x)\\&=\big(\overline{U}^{(1)},\overline{V}^{(1)}\big)(0,x).\end{aligned}$$

Since $h\in(0,1]$, for $x\in[-L,L]$ one easily checks that

$$\begin{aligned}&\big(\underline{U}^{(1)},\underline{V}^{(1)}\big)\big(\big((1-\delta)\rho\tau\big)^+,x\big)\\&=\big(h\underline{U}^{(0)}\big((1-\delta)\rho\tau,x\big),h\underline{V}^{(0)}\big((1-\delta)\rho\tau,x\big)\big)\\&\le\big(hu_1\big((1-\delta)\rho\tau+\tau,y\big),hu_2\big((1-\delta)\rho\tau+\tau,x\big)\big)\\&\le\big(h\overline{U}^{(0)}\big((1-\delta)\rho\tau+\tau,x\big),h\overline{V}^{(0)}\big((1-\delta)\rho\tau+\tau,x\big)\big),\end{aligned}$$

which is equal to

$$\begin{aligned}&\big(\underline{U}^{(1)},\underline{V}^{(1)}\big)\big(\big((1-\delta)\rho\tau\big)^+,x\big)\\&\le(u_1,u_2)\big(\big((1-\delta)\rho\tau\big)^++\tau,x\big)\\&\le\big(\overline{U}^{(1)},\overline{V}^{(1)}\big)\big(\big((1-\delta)\rho\tau\big)^+,x\big).\end{aligned}$$

By comparison arguments, we obtain

$$\big(\underline{U}^{(1)},\underline{V}^{(1)}\big)(t,x)\le(u_1,u_2)(t+\tau,x)\le\big(\overline{U}^{(1)},\overline{V}^{(1)}\big)(t,x)$$

for

$$t\in\big(0,(1-\delta)\rho\tau\big]\cup\big(\big((1-\delta)\rho\tau\big)^+,\tau\big],\quad x\in[-L,L],$$

and step by step, this inequality also holds for $t\ge 0$ and $x\in[-L,L]$.

By induction in n, we finally get

$$\big(\underline{U}^{(n)},\underline{V}^{(n)}\big)(t,x)\le(u_1,u_2)(t+n\tau,x)\le\big(\overline{U}^{(n)},\overline{V}^{(n)}\big)(t,x),\quad t\ge 0,\quad x\in[-L,L].$$

If $\lambda(\delta,H'(0),(-L,L))\ge 0$, recalling in Theorem 2.3(i) that the periodic problem (2.1) has no nonnegative nontrivial solution, so $\lim_{n\to+\infty}(\overline{U}^{(n)},\overline{V}^{(n)})=(0,0)$, which means that

$$\lim_{t\to+\infty}\|u_1(t,\cdot)\|_{\mathbb{C}[-L,L]}=\lim_{t\to+\infty}\|u_2(t,\cdot)\|_{\mathbb{C}[-L,L]}=0.$$

If $\lambda(\delta,H'(0),(-L,L))<0$, since

$$\lim_{n\to+\infty}\big(\underline{U}^{(n)},\underline{V}^{(n)}\big)(t,x)=\lim_{n\to+\infty}\big(\overline{U}^{(n)},\overline{V}^{(n)}\big)(t,y)=(U^*,V^*)(t,x)$$

in Theorem 2.3(ii), we finally arrive at that

$$\lim_{n\to+\infty}\big(u_1(t+n\tau,x),u_2(t+n\tau,x)\big)=\big(U^*(t,x),V^*(t,x)\big).$$

The proof is complete. □

4 Numerical simulations

In this section, we perform numerical simulations to illustrate the impact of a harvesting pulse and seasonal succession on species dynamics.

We firstly fix some parameters $d_1 = d_2 = 0.01, b = 0.8, \alpha_1 = 0.1, \alpha_2 = 0.2, a = 0.2, m_1 = 0.02, m_2 = 0.4, k_1 = 0.4, k_2 = 0.45, \tau = 50$ and $\rho = 0.5$.

In the following, we will choose various h and δ to investigate the effect arising from pulse intensity $(1-h)$ and duration of the bad season (δ).

We first fix $\delta = 0.5$ and let h vary to observe the impact of the harvesting pulse. Since juveniles u and adults v in (1.1) are cooperative, in the following we only show the figures concerning v.

It can be seen in Fig. 3 that increasing the harvesting pulse will speed up the process of extinction of species.

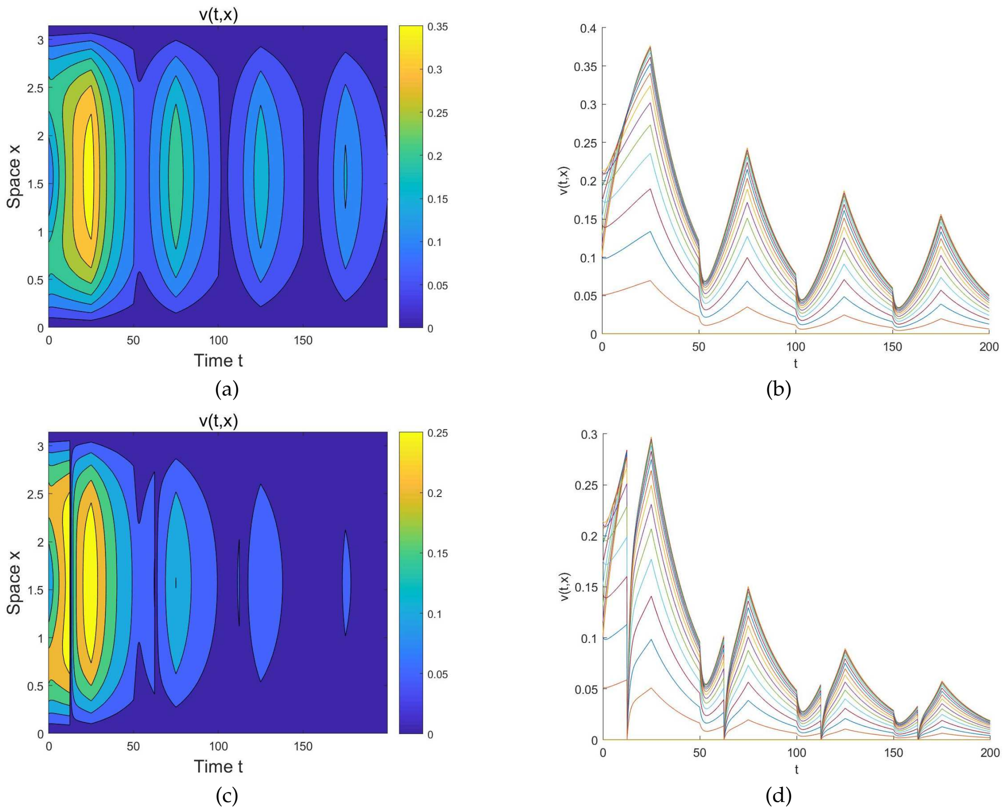
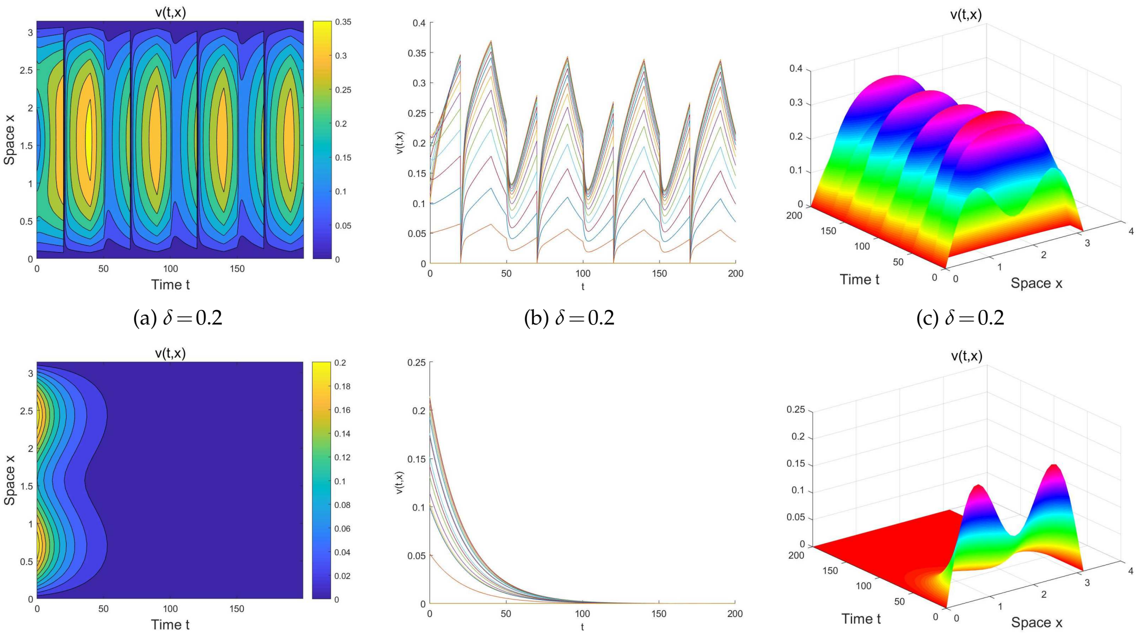


Figure 3: Fix $\delta = 0.5$. We firstly choose $h = 1$ in (a) and (b), which indicates that no pulse happens. Then $h = 0.1$ is fixed in (c) and (d), which means that the pulse intensity is $1-h = 0.9$. We see from (d) that, for $t \in [0,50]$, $(0,25]$ will be a good reason and (u,v) abides by reaction-diffusion equations, where the pulse occurs at $t = 12.5$, while $(25,50]$ is recognized as a bad season and (u,v) follows ordinary differential equations, see also $t \in (50,100]$ with harvesting time $t = 62.5$, $(100,150]$ with harvesting time $t = 112.5, \ldots$.

In the following, we will fix $h=0.1$ and choose different δ to investigate the effect of seasonal succession.

In comparison to Figs. 3(c)-3(d) and Figs. 4(a)-4(c), we observe that the increasing of δ will make species go from spreading to vanishing, which means that extending the duration of the bad season is harmful to the spreading of species. When $\delta=1$, only a bad season occurs, and the species will vanish, see Figs. 4(d)-4(f). One easily checks from Figs. 4(a)-4(c) with $\delta=0.2$, Figs. 3(c)-3(d) with $\delta=0.5$ and Figs. 4(d)-4(f) with $\delta=1$ that the larger the duration of the bad season is, the more unfavorable it is to the species, and it may even accelerate the extinction process of the species.



(a) $\delta=0.2$ (b) $\delta=0.2$ (c) $\delta=0.2$

(d) $\delta=1$ (e) $\delta=1$ (f) $\delta=1$

Figure 4: Fix $\delta=0.2$ in (a)-(c) and $\delta=1$ in (d)-(f), then keep other parameters unchanged as in Fig. 3. For (a)-(c), when $t\in(0,50]$, $t\in(0,40]$ is regarded as a good season and $t\in(40,50]$ is a bad season, see also $t\in(50,100],(100,150],(150,200],\cdots$. For this parameter, harvesting occurs at every time $t=20,70,120,\ldots$. The species finally stabilizes to a positive steady state. For (d)-(f), $\delta=1$ means only a bad season occurs, so (u,v) follows the ordinary differential equations and tends to zero.

Acknowledgments

The authors are grateful to the anonymous reviewers for careful reading and valuable comments that helped to improve the article.

The work is partially supported by the National Natural Science Foundation of China (Grant No. 12271470) and by the Yangzhou Innovation Capability Enhancement Fund (Grant No. YZ2024245).

References

[1] R. J. H. Beverton and S. F. Holt, *On the Dynamics of Exploited Fish Populations*, in: Great Britain, Fishery Investigations, Ser. II, Vol. XIX, H. M. Stationery Office, 1957.

[2] R. Canale and A. Vogel, *Effects of temperature on phytoplankton growth*, J. Environ. Eng. Div., 100:231–241, 1974.

[3] M. De la Sen and S. Alonso-Quesada, *Vaccination strategies based on feedback control techniques for a general SEIR-epidemic model*, J. Math. Biol., 218:3888–3904, 2011.

[4] D. L. DeAngelis, J. C. Trexler, and D. D. Donalson, *Competition dymamics in a seasonally varyingwetland*, in: Spatial Ecology, Chapman and Hall/CRC, 1–13, 2009.

[5] S. Gakkhar and K. Negi, *Pulse vaccination in SIRS epidemic model with non-monotonic incidence rate*, Chaos Solit. Fractals, 35:626–638, 2008.

[6] H. J. Guo, X. Y. Song, and L. S. Chen, *Qualitative analysis of a korean pine forest model with impulsive thinning measure*, Appl. Math. Comput., 234:203–213, 2014.

[7] S. Hsu and Y. Lou, *Single phytoplankton species growth with light and advection in a water column*, SIAM J. Appl. Math., 70:2942–2974, 2010.

[8] S. Hsu and X. Q. Zhao, *A Lotka-Volterra competition model with seasonal succession*, J. Math. Biol., 64:109–130, 2012.

[9] S. Hu and A. Tessier, *Seasonal succession and the strength of intra- and interspecific competition in a Daphnia assemblage*, Ecology, 76:2278–2294, 1995.

[10] C. Klausmeier, *Successional state dynamics: A novel approach to modeling nonequilibrium foodweb dynamics*, J. Theoret. Biol., 262:584–595, 2010.

[11] M. A. Lewis and B. T. Li, *Spreading speed, traveling waves, and minimal domain size in impulsive reaction-diffusion models*, Bull. Math. Biol., 74:2383–2402, 2012.

[12] L. Y. Li and N. K. Sun, *A free boundary problem with advection and seasonal succession*, J. Differential Equations, 423:512–549, 2025.

[13] Y. Li and H. Y. Zhao, *On a reaction-diffusion hybrid mosquito model with impulsive control and imperfect maternal transmission of Wolbachia*, J. Nonlinear Sci., 35:29, 2025.

[14] G. M. Lieberman, *Second Order Parabolic Differential Equations*, World Scientific Publishing Co. Inc., 1996.

[15] Y. Meng, Z. G. Lin, and M. Pedersen, *Effects of impulsive harvesting and an evolving domain in a diffusive logistic model*, Nonlinearity, 34:7005–7029, 2021.

[16] C. V. Pao, *Stability and attractivity of periodic solutions of parabolic systems with time delays*, J. Math. Anal. Appl., 304:423–450, 2005.

[17] R. Peng and X. Q. Zhao, *The diffusive logistic model with a free boundary and seasonal succession*, Discrete Contin. Dyn. Syst., 33:2007–2031, 2013.

[18] L. Q. Pu, Z. G. Lin, and Y. Lou, *A West Nile virus nonlocal model with free boundaries and seasonal succession*, J. Math. Biol., 86:25, 2023.

[19] R. Q. Shi, X. W. Jiang, and L. S. Chen, *The effect of impulsive vaccination on an SIR epidemic model*, Appl. Math. Comput., 212:305–311, 2009.

[20] X. Y. Song and Y. F. Li, *Dynamic complexities of a Holling II two-prey one-predator system with impulsive effect*, Chaos Solit. Fractals, 33:463–478, 2007.

[21] C. F. Steiner, A. S. Schwaderer, V. Huber, C. A. Klausmeier, and E. Litchman, *Periodically forced food-chain dynamics: Model predictions and experimental validation*, Ecology, 90:3099–3107, 2009.

[22] S. Y. Tang and R. A. Cheke, *State-dependent impulsive models of integrated pest management (IPM) strategies and their dynamic consequences*, J. Math. Biol., 50:257–292, 2005.

[23] X. L. Tong and Z. G. Lin, *Analysis of mosquito diffusion model with pulse intervention*, Journal of Yangzhou University (Natural Science Editor), 28:1–12, 2025. (in Chinese)

[24] R. W. Wu and X. Q. Zhao, *Spatial invasion of a birth pulse population with nonlocal dispersal*, SIAM J. Appl. Math., 79:1075–1097, 2019.

[25] H. Y. Xu, Z. G. Lin, and C. A. Santos, *Interaction between harvesting intervention and birth perturbation in an age-structured model*, J. Math. Biol., 91:12, 2025.

[26] Y. Xue, S. Y. Tang, and J. H. Liang, *Optimal timing of interventions in fishery resource and pest management*, Nonlinear Anal. Real World Appl., 13:1630–1646, 2012.