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PROVABLY CONVERGENT PLUG-AND-PLAY PROXIMAL BLOCK COORDINATE DESCENT METHOD FOR HYPERSPECTRAL ANOMALY DETECTION

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Abstract. Hyperspectral anomaly detection refers to identifying pixels in the hyperspectral images that have spectral characteristics significantly different from the background. In this paper, we introduce a novel model that represents the background information using a low-rank representation. We integrate an implicit proximal denoiser prior, associated with a deep learning based denoiser, within a plug-and-play (PnP) framework to effectively remove noise from the eigenimages linked to the low-rank representation. Anomalies are characterized using a generalized group sparsity measure, denoted as $\|\cdot\|_{2,\psi}$. To solve the resulting orthogonal constrained nonconvex nonsmooth optimization problem, we develop a PnP-proximal block coordinate descent (PnP-PBCD) method, where the eigenimages are updated using a proximal denoiser within the PnP framework. We prove that any accumulation point of the sequence generated by the PnP-PBCD method is a stationary point. We evaluate the effectiveness of the PnP-PBCD method on hyperspectral anomaly detection in scenarios with and without Gaussian noise contamination. The results demonstrate that the proposed method can effectively detect anomalous objects, outperforming the competing methods that may mistakenly identify noise as anomalies or misidentify the anomalous objects due to noise interference.

Key words. Low-rank representation, proximal block coordinate descent, hyperspectral anomaly detection, plug-and-play.

1. Introduction

Hyperspectral anomaly detection aims to identify pixels or regions in hyperspectral images (HSIs) that significantly differ from the surrounding background without prior knowledge of the target spectral information. These pixels, often referred to as anomalies, could represent objects or materials such as aircraft, ships, vehicles, or other structures that deviate from the natural background. Detecting such anomalies is crucial due to their significance in various applications. For example, in environmental monitoring, anomalies may indicate areas affected by pollution or disease in vegetation [22]; in the food industry, anomalies may be detected for quality control by identifying physical defects and inconsistencies in products [28]. By leveraging the rich spectral information provided by HSIs, the accuracy and reliability of anomaly detection can be enhanced, thereby improving decision-making processes in fields such as security, agriculture, and resource management.

In hyperspectral anomaly detection, the Reed-Xiaoli (RX) method, introduced by Reed and Xiaoli in 1990 [23], is a foundational method known for its simplicity and widespread adoption. The RX method assumes that background spectral features follow a multivariate Gaussian distribution and identifies anomalies by calculating the Mahalanobis distance from the background. Over time, RX has inspired several variants to address its limitations in real-world applications. For example, the local RX method [20] enhances localized anomaly detection using sliding windows for background estimation; the kernel RX method [17] maps data into

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high-dimensional feature spaces to better adapt to nonlinear distributions; and the weighted RX method [10] introduces pixel-level weighting for improving robustness against noise. While RX and its variants are computationally efficient and serve as benchmarks in the field, they often rely on Gaussian assumptions and are sensitive to noise and outliers, limiting their performance in complex scenes.

In contrast to statistical approaches like RX, representation-based methods focus on explicitly modeling the structure of HSIs without assuming a predefined distribution. Li et al. [18] proposed the background joint sparse representation detection (BJSRD) method, which reconstructs each background pixel using a s-parse set of coefficients from a dictionary. Xu et al. [31] introduced the low-rank and sparse representation (LRASR) method, which models the background as a low-rank component while representing anomalies as sparse components. Feng et al. [7] developed the local spatial constraint and total variation (LSC-TV) method, which combines low-rank modeling with superpixel segmentation and total variation (TV) regularization to effectively separate anomalies in complex scenes. To preserve the intrinsic 3D structure of HSIs, the low-rank component is characterized using tenor low-rank representation. For example, the tensor low-rank and sparse representation (TLRSR) method [26] utilizes the tensor singular value decomposition (t-SVD), while the method proposed in [6] employs the tensor ring decomposition.

Deep learning methods have significantly improved hyperspectral anomaly detection by extracting hierarchical features from high-dimensional data using deep neural networks. Among these, the Auto-AD method [27], a fully convolutional autoencoder, autonomously reconstructs the background and highlights anomalies through reconstruction errors, eliminating the need for manual parameter tuning or preprocessing. Other neural network models, such as stacked denoising autoencoders (SDAs) [35] and spectral-constrained adversarial autoencoders (SC-AAE) [30], use manifold learning and adversarial strategies to enhance anomaly detection capabilities. These approaches are highly effective in nonlinear and complex environments but often require large datasets and significant computational resources, which can pose challenges for real-time applications.

In this paper, we develop a novel approach for hyperspectral anomaly detection that utilizes a representation-based technique for expressing the background, a deep learning denoiser for reducing noise contamination and a group sparsity measure for identifying anomalies. Our main contributions are summarized as follows:

- We represent the background of HSIs in terms of a tensor mode-3 product of a learnable orthogonal basis as the subspace and a tensor formed by eigenimages as the representation coefficients.
- We employ a deep learning denoiser in a plug-and-play (PnP) fashion to eliminate the noise from the eigenimages. We enhance the existing relaxed proximal denoiser to its shifted version to denoise the eigenimages that may not fall within the pretrained range. The proposed denoiser can also be viewed as a proximal operator associated with a weakly convex function.
- We introduce a generalized group sparsity measure, $\|\cdot\|_{2,\psi}$, to detect sparse anomalous objects. The function ψ is a sparsity-promoting function and can be chosen as a weakly convex function.
- We propose a PnP version of the proximal block coordinate descent algorithm, called the PnP-PBCD method, for solving the proposed nonconvex nonsmooth minimization problem with an orthogonal constraint. The subproblems have either closed-form solutions or are easy to compute. We