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PHYSICS-BASED MULTI-TIME STEPPING ALGORITHM FOR THREE-DIMENSIONAL QUASI-STATIC ELECTROPOROELASTICITY EQUATIONS

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Abstract. Electroporoelasticity equations comprise of Maxwell's equations and Biot's equations, playing an important role in geophysical areas such as oil-gas exploration and earthquake early warning. The development of electromagnetic waves and elastic waves presents distinct time scales due to the multi-physics nature. In this paper, we propose a multi-time stepping numerical algorithm to approximate electroporoelasticity equations, in which we use a smaller time step to compute Maxwell's equations and a larger time step to calculate Biot's equations. We prove the stability of this algorithm and derive its error estimates. Numerical experiments are conducted to demonstrate the theoretical analysis.

Key words. Electroporoelasticity equations, multi-time stepping algorithm, finite element method, error estimates.

1. Introduction

Natural resource reservoirs such as water, oil and gas are predominantly poroelastic media, comprising of solid skeletons and pore fluids [7]. The seismoelectric coupling phenomenon emerges from the relative motion between the solid matrix and fluid in fluid-saturated porous media, induced by the presence of an electric double layer consisting of an adsorbed layer and a diffuse layer. This phenomenon includes both the seismoelectric effect and the electroseismic effect and leads to seismoelectric coupling waves [25]. In the absence of fluid flow, the porous medium is electrically neutral overall. However, when seismic waves propagate through the porous medium, fluid flow occurs within the pores, causing charges in the diffuse layer to move relatively to those in the adsorbed layer and thereby forming a streaming current, which is the seismoelectric effect. And vice versa, the electroseismic effect occurs when the electric field in the porous medium changes, charges in the diffuse layer move within the electric field to generate a conduction current, which simultaneously drags the fluid in the diffuse layer into motion.

Seismoelectric coupling is modeled by electroporoelasticity equations which consist of Maxwell's equations [2] and Biot's equations via an electrokinetic coupling coefficient. Seismoelectric coupling waves integrate the spatial resolution of elastic waves with the reservoir identification capability of electromagnetic waves, enabling the seismoelectric coupling effect to find applications in diverse fields such as oil-gas exploration [35], earthquake early warning [26, 27], environmental protection [20], water conservancy exploration [15], and other areas of geophysics [3, 34]. Due to its importance in applications, much attention has already been paid to electroporoelasticity equations, cf. [11, 12, 13, 16, 18, 19, 29, 30].

Numerical methods are important tools to explore the multi-physics nature of electroporoelasticity equations. Hu and Meir [12] proposed a numerical scheme using the standard finite element method (FEM) and analyzed its error estimates.

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Liu et al. [16] investigated the well-posedness and applied splitting technique to set up a finite element approximation to improve the computational efficiency. In recent years, a multi-time stepping technique has been developed to accelerate the numerical computations for solving partial differential equations. Shan et al. [31] constructed a decoupled scheme with different time steps for a nonstationary Stokes-Darcy model and verified its stability and convergence. Rybak and Magiera [28] developed a mass conservative multi-time stepping method for solving coupled free flow and porous medium flow problems, proved its long time stability and performed error eatimates. Shevchenko et al. [32] presented a multi-time stepping integration method for the ultrasound heating problem and showed its efficiency and robustness. Zhang et al. [38] studied a finite element approximation to the Stokes-Darcy-Transport system with different time steps on different physical variables. In these studies, the large discrepancy in time scales of partial differential equations leads to that the equations can be solved with different time scales. The key idea is to use a small time step to discretize the temporal variable in the equation that changes rapidly, while using a large time step to solve the equation that changes slowly. For more references, we refer to [8, 14, 24, 33, 39] and the literature therein.

Electroporoelasticity equations form a complex system of coupled, multiphysics, multi-component, and multiscale models. It is common knowledge that electromagnetic waves propagate much faster than seismic waves. In this paper, we investigate a multi-time stepping algorithm to improve the efficiency of numerical approximations to three-dimensional quasi-static electroporoelasticity equations. The main idea consists of two steps. First, we decouple the electroporoelasticity equations into Maxwell's equations and Biot's equations. Then, we discretize Maxwell's equations with a smaller time step-size and approximate Biot's equations with a larger time step-size. We prove the stability and first-order convergence in temporal and spatial variables, respectively, of the multi-time stepping algorithm.

The rest of this paper is organized as follows. Section 2 introduces the quasi-static electroporoelasticity equations along with the finite element spaces. In Section 3, we propose a physics-based multi-time stepping algorithm and establish its stability. Error estimates for the numerical approximation are derived in Section 4. Section 5 presents numerical experiments that validate the theoretical analysis.

2. Preliminaries

In this section, we describe quasi-static electroporoelasticity equations and introduce finite element spaces used in this paper.

2.1. Electroporoelasticity equations. Let $[0,T]$ with $T>0$ be an interval and $\mathscr{D}\subset\mathbb{R}^3$ be an open bounded polyhedron with $\mathbf{n}$ the unit outward normal vector to boundary $\partial\mathscr{D}$. Consider three-dimensional quasi-static electroporoelasticity equations for $(t,\mathbf{x})\in[0,T]\times\mathscr{D}$

$$
\begin{aligned}
&\epsilon\frac{\partial}{\partial t}\mathbf{E}+\sigma\mathbf{E}-\nabla\times\mathbf{H}-L\nabla p=\mathbf{j},\\
&\mu\frac{\partial}{\partial t}\mathbf{H}+\nabla\times\mathbf{E}=\mathbf{0},\\
&-(\lambda+G)\nabla(\nabla\cdot\mathbf{u})-G\Delta\mathbf{u}+\alpha\nabla p=\mathbf{f},\\
&\frac{\partial}{\partial t}(c_0p+\alpha\nabla\cdot\mathbf{u})-\kappa\Delta p+L\nabla\cdot\mathbf{E}=g,
\end{aligned}
\tag{1}
$$

with initial value conditions for $\mathbf{x}\in\mathscr{D}$

$$
\mathbf{E}(0,\mathbf{x})=\mathbf{E}_0(\mathbf{x}),\quad \mathbf{H}(0,\mathbf{x})=\mathbf{H}_0(\mathbf{x}),\quad \mathbf{u}(0,\mathbf{x})=\mathbf{u}_0(\mathbf{x}),\quad p(0,\mathbf{x})=p_0(\mathbf{x}),
\tag{2}
$$