J. At. Mol. Sci.
doi: 10.4208/jams.032610.042010a

Vol. **1**, No. 4, pp. 344-351
November 2010

Remote preparation of three-particle GHZ-class states

Gui-Xia Pan*

School of Science, Anhui University of Science and Technology, Huainan 232001, China

Received 26 March 2010; Accepted(in revised version) 20 April 2010;
Published online 2 August 2010

Abstract. A scheme is presented for remote state preparation of three-particle GHZ-class states by using a Bell state and an asymmetric W state as the quantum channel. In the scheme, the success probability of preparation and classical communication cost are calculated. In general, Bob can successfully prepare the initial state with the probability 1/4 and consume 1/4 classical bits. However, in special situations the success probability of preparation can reach 1/2 or even 1 after consuming a little additional classical bits.

PACS: 03.67.Hk, 03.67.-a, 03.65.-w

Key words: remote state preparation, asymmetric W state, two-qubit projective measurement, classical communication, unitary operation

1 Introduction

In quantum information field, quantum entanglement and classical communication are two elementary resources. Bennett *et al*. [1] first proposed quantum teleportation protocol in 1993. In the scheme, an arbitrary unknown quantum state can be teleported by utilizing a prior shared entanglement and some classical communication. Very similar to quantum teleportation [2–7], a distinct application of quantum entanglement, i.e., remote state preparation (RSP) is first presented by Lo [8] in 2000. The main common point between teleportation and RSP is that entanglement should inhabit the quantum channel linking two parties. In contrast, the key difference between usual teleportation and RSP is that, in RSP the preparer Alice is assumed to completely know the state to be prepared, while in teleportation schemes the sender Alice needs not to know the state to be transmited.

The principal concern of RSP is to study whether the required classical communication and entanglement cost can be reduced in the case that the sender Alice knows

*Corresponding author. *Email address:* gxpan@aust.edu.cn (G. X. Pan)

the prepared state. So far one has already known that, for general states RSP does not overwhelm quantum teleportation due to its probabilistic success in preparation. However, for some special ensembles of states, Pati [9] has found RSP protocol is more economical than teleportation. Since then, RSP has already attracted many attentions and various kinds of theoretical RSP protocols have been proposed [10–24], such as low-entanglement RSP [10], higher-dimension RSP [11], optimal RSP [12], oblivious RSP [13], RSP without oblivious conditions [14], generalized RSP [15], faithful RSP [16], RSP for multiparties [17], and continuous variable RSP in phase space [18], etc. Meantime, RSP schemes have been implemented experimentally by using Nuclear magnetic resonance (NMR) [25] and spontaneous parametric down-conversion [26].

2 The RSP scheme with a Bell state and asymmetric W state

In this paper, we will present a protocol for remotely preparing a three-particle GHZ-class state by using a Bell state and a asymmetric W state as the quantum channel. In the scheme, suppose that Alice wants to help Bob remotely prepare a three-particle GHZ-class state. The GHZ class state $|M\rangle$ is written as

$$|M\rangle = q|000\rangle + r|111\rangle + u|001\rangle + v|110\rangle, \tag{1}$$

where q, r, u and v are complex and satisfy $|q|^2+|r|^2+|u|^2+|v|^2=1$. Bob does not know the coefficients but Alice does. The prior established quantum channel consisting of a Bell state and a asymmetric W state is represented as

$$|\varphi\rangle_{a_1b_1} = \frac{1}{\sqrt{2}}|01\rangle_{a_1b_1} + \frac{1}{\sqrt{2}}|10\rangle_{a_1b_1}, \tag{2}$$

$$|\varphi\rangle_{a_2b_2b_3} = \frac{1}{2}|001\rangle_{a_2b_2b_3} + \frac{1}{2}|010\rangle_{a_2b_2b_3} + \frac{1}{\sqrt{2}}|100\rangle_{a_2b_2b_3}. \tag{3}$$

Obviously, the total state of the five-qubit system can be expressed as

$$|\psi\rangle_{a_1b_1a_2b_2b_3} = |\varphi\rangle_{a_1b_1} \otimes |\varphi\rangle_{a_2b_2b_3}. \tag{4}$$

It is assumed that Alice owns qubits a_1 and a_2, and b_1, b_2 and b_3 belong to Bob. To prepare the state $|M\rangle$ in Bob's site, Alice first carries out a two-qubit projective measurement on her qubit pair (a_1,a_2) in the mutually orthogonal basis vectors $\{|P_1\rangle, |P_2\rangle, |P_3\rangle, |P_4\rangle\}$. These vectors are defined as

$$|P_1\rangle = q|00\rangle + r|11\rangle + u|01\rangle + v|10\rangle, \tag{5}$$

$$|P_2\rangle = \omega q|00\rangle + \omega r|11\rangle - \omega^{-1}u|01\rangle - \omega^{-1}v|10\rangle, \tag{6}$$

$$|P_3\rangle = r^*|00\rangle - q^*|11\rangle + v^*|01\rangle - u^*|10\rangle, \tag{7}$$

$$|P_4\rangle = \omega r^*|00\rangle - \omega q^*|11\rangle - \omega^{-1}v^*|01\rangle + \omega^{-1}u^*|10\rangle, \tag{8}$$