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Some Improvements in Preconditioned Modified Accelerated Overrelaxation (PMAOR) Method for Solving Linear Systems

Hashem Saberi Najafi, Seyyed Ahmad Edalatpanah[+]

Department of Mathematics, Faculty of Sciences, Islamic Azad University of Lahijan, P. Code 1616,Lahijan, Iran

Department of Mathematics, Faculty of Sciences, The University of Guilan, Rasht, Iran, P.O.box 41335-1914

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Abstract. In this article a new preconditioner from class of (I+S)-type based on the Modified Accelerated Overrelaxation(MAOR) iterative method has been introduced and convergence properties of the proposed method have been analyzed and compared with the some other preconditioners. Moreover, comparisons between different splittings are derived. Numerical example is also given to illustrate our results.

Keywords: preconditioning; Comparison theorems; MAOR method; Splitting; M-matrix

1. Introduction

Science history indicates that substantial improvements and huge jumps in science and technology require interaction between mathematicians with different scientists. Meanwhile, solving linear equation system play the role of a catalyst for further connection of this interaction between mathematics and sciences.

Consider the following linear system

$$AX=b \tag{1.1}$$

Where $b, X \in R^n$ and $A \in R^{n\times n}$ is an nonsingular matrix of the following block form

$$A=\begin{bmatrix} D_1 & H \\ K & D_2 \end{bmatrix} \tag{1.2}$$

Also D_1,D_2 are nonsingular diagonal matrices of orders n_1 and n_2 respectively and $H \in R^{n_1\times n_2}$,$K \in R^{n_2\times n_1}$.
For any splitting, A=M-N with det(M) ≠0, the basic iterative methods for solving(1.1) is

$$X^{(t+1)} = M^{-1}NX^{(t)} + M^{-1}b \qquad t=1,2,\ldots \tag{1.3}$$

This iterative process converges to the unique solution $X = A^{-1}b$ for any initial vector value $X_0 \in R^n$ if and only if the spectral radius $\rho(M^{-1}N)<1$,where $M^{-1}N$ is called the iterative matrix. There are some specifically iterative methods for solving a linear system (1.1) based on (1.3).see[1].

Modified Overrelaxation methods are also above model .These methods have been discussed and used by many researchers; see [1-7]. Let the matrix **A** have the splitting **A**=D -C_L-C_u=D(I-L-U),where $L = D^{-1}C_L$, $U = D^{-1}C_u$, D = diag(A), C_L and C_u are strictly lower and upper triangular matrices of **A,** respectively. The modified accelerated Overrelaxation (MAOR) method defined by [4] is

$$X^{(t+1)} = \mu_{\Omega,\Gamma}X^{(t)} + \Psi \qquad t=0,1,\cdots \tag{1.4}$$

With iterative matrix

[+] Corresponding author. *E-mail address*: saedala@yahoo.com & saedalatpanah@gmail.com

$$\mu_{\Omega,\Gamma} = M^{-1}N = \underbrace{(D-\Gamma C_L)^{-1}}_{M^{-1}}\underbrace{[(I-\Omega)D+(\Omega-\Gamma)C_L+\Omega C_U]}_{N}$$
$$= \underbrace{(I-\Gamma L)^{-1}}_{M^{-1}}\underbrace{[(I-\Omega)+(\Omega-\Gamma)L+\Omega U]}_{N} \tag{1.5}$$

And

$$\Psi = (D-\Gamma C_L)^{-1}\Omega b = (I-\Gamma L)^{-1}D^{-1}\Omega b \tag{1.6}$$

With

$$\Omega = diag(w_1 I_1, w_2 I_2), \Gamma = diag(\gamma_1 I_1, \gamma_2 I_2) \tag{1.7}$$

Where $w_1, w_2, \gamma_1, \gamma_2$ are positive real parameters and I_1, I_2 are identity matrix of orders n_1 and n_2 respectively. Darvishi et al. in [8] studied preconditioned MAOR method for linear systems based on preconditioners of class (I+S)-type (For details, we refer to [9-21]). They proposed following splitting of A

$$D = \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}, C_L = \begin{bmatrix} 0 & 0 \\ -K & 0 \end{bmatrix}, C_U = \begin{bmatrix} D_1^* & -H \\ 0 & D_2^* \end{bmatrix} \tag{1.8}$$

Where

$$D_1^* = I_1 - D_1, D_2^* = I_2 - D_2 \tag{1.9}$$

They assume that

$$H \le 0, \quad K \le 0, \quad 0 \le w_1 \le 1, \quad 0 \le w_2 \le 1, \quad 0 \le \gamma_2 \le \frac{w_2}{w_1} \tag{1.10}$$

Also they presented following preconditioners **P**of**A**,

Where

$$P = (D + \bar{S}) \tag{1.11}$$

With

$$\bar{S} = \begin{bmatrix} 0 & 0 \\ s_i & 0 \end{bmatrix} \qquad i = 1,2,3 \tag{1.12}$$

Where

$$S_1 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{K_{n_2,1}}{\alpha} & 0 & \cdots & 0 \end{bmatrix}, \alpha > 0 \tag{1.13}$$

$$S_2 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ -K_{11} & 0 & \cdots & 0 \\ -K_{21} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -K_{n_2,1} & 0 & \cdots & 0 \end{bmatrix} \tag{1.14}$$

And

$$S_3 = \begin{cases} -K_{i,j} & for \quad i = j, i = 1,2,\ldots,n_2 \\ 0 & for \quad otherwise \end{cases} \tag{1.15}$$

Moreover, they showed that the preconditioned matrix

$$\bar{A} = PA \tag{1.16}$$