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Exact Solutions for Some Nonlinear Partial Differential Equations in Mathematical Physics

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Abstract. In this article, by introducing a new general ansatze, the improved (G'/G)- expansion - method is proposed to construct exact solutions of some nonlinear partial differential equations in mathematical physics via the generalized Zakharov equations, the coupled Maccaris equations, the (2+1)-dimensional Wu-Zhang equations and the (1+1) dimensional Fornberg – Whitham equation in terms of the hyperbolic functions , trigonometric functions and rational function, where G satisfies a second order linear ordinary differential equation. When the parameters are taken special values, the solitary wave are derived from the traveling waves. This method is reliable, simple and gives many new exact solutions.

Keywords: The improved (G'/G)- expansion method, Traveling wave solutions, The generalized Zakharov equations, The coupled Maccaris equations, The (1+1) dimensional Fornberg – Whitham equation , The (2+1)-dimensional Wu-Zhang equations.

1. Introduction

Nonlinear partial differential equations are known to describe a wide variety of phenomena not only in physics, where applications extend over magneto fluid dynamics, water surface gravity waves, electromagnetic radiation reactions, and ion acoustic waves in plasma, just to name a few, but also in biology and chemistry, and several other fields. It is one of the important tasks in the study of the nonlinear partial differential equations to seek exact and explicit solutions. In the past several decades both mathematicians and physicists have made many attempts in this direction. Various methods for obtaining exact solutions to nonlinear partial differential equations had been proposed. Among these are the inverse scattering method [1], Hirota's bilinear method [2], Backlund transformation [3,4], Painlevé expansion [5], sine–cosine method [6], homogenous balance method [7], homotopy perturbation method [8–11], variation method [12,13], Adomian decomposition method [14,15], tanh - function method [16–18], Jacobi elliptic function expansion method [19–22], F-expansion method [23–25] and Exp-function method [26–28].

Wang etal [29] proposed a new method called the (G'/G) expansion method to look for the traveling wave solutions for nonlinear partial differential equations (NPDEs) . By using the (G'/G) expansion method, Zayed etal [30,31] and the modified (G'/G) expansion method, Shehata [32] have successfully obtained more traveling wave solutions for some important NPDEs. Recently Guo etal [33] had developed the (G'/G) expansion method for solving the NPDEs. In this paper we use the improvement (G'/G) expansion method to find the traveling wave solutions for the generalized Zakharov equations, the coupled Maccaris equations, the (2+1)-dimensional Wu-Zhang equations and the (1+1) dimensional Fornberg – Whitham equation.

2. Description of the improvement (G'/G) expansion method for NPDEs

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In this section, we give the detailed description of our method. Suppose that a nonlinear evolution equation, say in two independent variables x and t is given by

$$P(u,u_t,u_x,u_{tt},u_{xx},u_{xt},..)=0, \quad (1)$$

where $u=u(x,t)$ is an unknown function, P is a polynomial in $u=u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved.

To determine u explicitly, we take the following five steps [33]:

Step 1: We use the following travelling wave transformation:

$$u=U(\xi), \qquad \xi=x-kt, \quad (2)$$

where k is a constant to be determined latter. The NPDE (1) is reduced to a nonlinear ordinary differential equation (NODE) in $U(\xi)$:

$$P(U,U',U'',\ldots)=0\ . \quad (3)$$

Step 2. We suppose the following series expansion as a solution of Eq. (3):

$$U(\xi)=\sum_{i=-m}^{m}\frac{\alpha_i\left(\frac{G'(\xi_n)}{G(\xi_n)}\right)^i}{\left[1+\sigma\left(\frac{G'(\xi_n)}{G(\xi_n)}\right)\right]^i}, \quad (4)$$

where $\alpha_i\,(i=0,\pm1,\ldots,\pm m)$, σ are constants to be determined later, m is a positive integer and $G(\xi)$ satisfies a second order linear ordinary differential equation

$$G''(\xi)+\mu G(\xi)=0, \quad (5)$$

where μ is a real constants. The general solutions of Eq. (5), can be listed as follows. When $\mu<0$, we obtain the hyperbolic function solution of Eq.(5)

$$G(\xi)=C_1\cosh(\sqrt{-\mu}\xi)+C_2\sinh(\sqrt{-\mu}\xi). \quad (6)$$

When $\mu>0$, we obtain the trigonometric function solution of Eq.(5)

$$G(\xi)=C_1\sin(\sqrt{\mu}\xi)+C_2\cos(\sqrt{\mu}\xi)\ . \quad (7)$$

When $\mu=0$, we obtain the rational function solution of Eq.(5)

$$G(\xi)=C_1\xi+C_2\,. \quad (8)$$

where C_1 and C_2 are arbitrary constants.

Step 3. Determine the positive integer m by balancing the highest order nonlinear term(s) and the highest order derivative in Eqs. (1) or (3).

Step 4. Substituting Eq. (4) along with (5) into (3) , cleaning the denominator and then setting all the coefficients of $(G'(\xi)/G(\xi))^i, i=0,\pm1,\pm2,..$ to be zero, yield a set of algebraic equations for which the constants $\alpha_i\,(i=0,\pm1,\ldots,\pm m)$, k and σ .

Step 5. Assuming that the constants $\alpha_i\,(i=0,\pm1,\ldots,\pm m)$, k and σ can be obtained by solving the algebraic equations in Step 4, then substituting these constants and the known general solutions of Eq. (5) into (4), we can obtain the explicit solutions of Eq. (1) immediately.

3. Applications of the improved (G'/G) expansion method for NPDEs

In this section, we apply the improved (G'/G)- expansion method to construct the traveling wave