

Application of an Ant Colony System – Node (ACS – N) algorithm in the Vehicle Routing Problem (VRP)

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Abstract. Ant colony Optimization (ACO) is a relatively new class of metaheuristic search techniques for hard optimization problems. In this paper we focus on the definition and minimization of the objective function of the VPR using an Ant Colony System – Node (ACS – N) algorithm. The (ACS – N) algorithm is implemented for an eight node graph with respective demands. Moreover, in this paper we study the effect of the number of the ants to the value of the objective function.

Keywords: Ant Colony Optimization (ACO), Vehicle Routing Problem (VRP), Ant System (AS).

1. Introduction

The Vehicle Routing Problem (VRP), which was introduced by Dantzig and Ramser [1], is an important combinatorial optimization problem in the field of operations management and logistics.

The VRP is an NP-hard problem, and can be described as follows: items are to be delivered to a set of customers by a fleet of vehicles from a common depot. The locations of the customers and the depot are given. The aim is to determine a set of vehicle routes:

- i) of minimum total cost,
 - ii) starting and ending at the common depot,
- and
- iii) each customer is served exactly once by exactly one vehicle,
 - iv) the total duration of each route must not exceed the constraint
 - v) the total demand of any route does not exceed the capacity of the vehicle

A number of researchers developed heuristics techniques to solve the VRP such as tabu search (TS) [2], simulated annealing (SA) [3] and genetic algorithms (GA) [4].

More recently Ant Colony Optimization (ACO) algorithm has been evolved for solving difficult Optimization problems.

ACO is a new Metaheuristic that is based on the foraging behaviour of the real ants. One of the main ideas behind this approach is that the ants can communicate with one another through indirect means by making modification to the concentration of highly volatile chemical called pheromones in their immediate environment [5].

In this paper we present the Ant Colony System – Node (ACS-N) algorithm through which we control the pheromone. In most cases, the structure of the problem described with the use of a graph. The use of the (ACS-N) algorithm demands the pheromone to be placed at the nodes of the graph saving memory of $O(n^2)$ comparing with the classic structure of the pheromone that it is used at the basic algorithm ACO (AS, ACS, Max – Min Ant System).

The paper has the following structure: in the section 2 it is presented a sort introduction in the Ant Colony optimization and especially we insert a stochastic model, concerning the pheromone distribution through difference equations.

Section 3 we give the problem formulation of the VRP problem. Section 4 we present the mathematical model of (ACS-N) algorithm. Section 6 we present the case study. Finally in section 7 we draw some conclusions.

2. ANT COLONY OPTIMIZATION

The Ant Colony Optimization (ACO) was proposed by Dorigo and co-authors [6] use artificial ants, called agents, to solving various Combinatorial Optimization problems. The artificial ants to mimic the cooperative of real ants behaviour, plus additional capabilities that make them more effective, such as a memory of past actions.

Generally, an ant walking from the nest to food sources and vice versa deposits on the ground a chemical substance called pheromone, forming a pheromone trail.

We suppose that there are two possible paths to reach a food source, as shown in Fig. 1. Also, supposing M ants cover per second the paths, to every direction with constant velocity $u \text{ cm/s}$, and deposit a unit of pheromone in each path. In this paper the evaporation of pheromone is neglect.

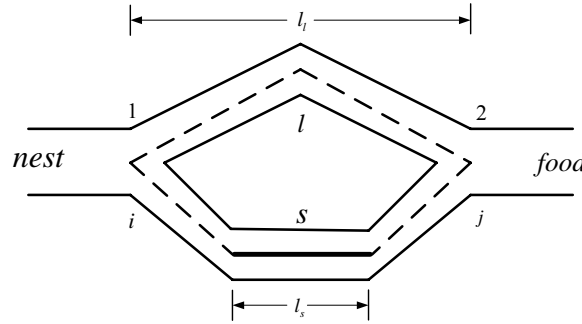


Figure 1. The pheromone deposition of ants

If l_s and l_l are the lengths in cm, of the sorter (s) and longer (l) path, and t_s , t_l respectively.

Is the time that an ant choose to cover the sorter (s) and longer (l) path, then the probability $P_{ia}(t)$ for an ant to reach the decision point $i \in \{1,2\}$ and choose the path $a \in \{s,l\}$ is [7]:

$$P_{ia} = \frac{(t_s + \Phi_{is}(t))^a}{(t_s + \Phi_{is}(t))^a + (t_s + \Phi_{il}(t))^a} \quad (1)$$

Where $\Phi_{ia}(t)$ is the total amount of pheromone on the paths, which is proportional to the number of ants use the paths in time t . experimentally to accept $\alpha=2$.

The amount of pheromone $\Phi_{is}(t)$ (respectively $\Phi_{il}(t)$) which is deposited on the S path (respectively l) is given by solution of the following difference equation system [8]:

$$\Phi_{is}(t+1) - \Phi_{is}(t) = M[P_{js}(t-t_s) + P_{is}(t)] \quad (2)$$

$$\Phi_{il}(t+1) - \Phi_{il}(t) = M[P_{jl}(t-t_s) + P_{il}(t)] \quad (3)$$

In fact, the difference equations (2), (3) are of the first degree, non homogenous.

To point $\Phi_{is}(t-1) \equiv y_{t+1}$, $\Phi_{is}(t) \equiv y_t$ and $M[P_{js}(t-t_s) + P_{is}(t)] \equiv g_t$. The difference equation (2) to transform:

$$y_{t+1} - y_t \equiv g_t \quad (4)$$

The general solution is: $y_t = k + \sum_{\lambda=1}^{t-1} g_\lambda$

$$\text{or } y_t = k + \sum_{\lambda=1}^{t-1} M[P_{js}(t-t_s) + P_{is}(t)]$$

$$\text{or } \Phi_{is}(t) = k + \sum_{\lambda=1}^{t-1} M[P_{js}(t-t_s) + P_{is}(t)] \quad (5)$$

Similarly the solution of the difference equation (3) is: