ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 11, No. 4, 2016, *pp*.291-297

Control and Stability of the Time-delay Linear Systems

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(Received Jun 04, 2016 , accepted August 18, 2016)

Abstract. This paper presents a method for control discrete-time system with time-delay. The main idea is a convert the discrete-time delay linear controllable system into the linear systems without delay. Then by using similarity transformations a state feedback matrix was obtained, so that time-delay has no effect on the system. The proposed technique is illustrated by means of numerical examples.

Keywords: stability, Discrete-time, Time-delay, State feedback matrix, Pole Assignment.

1. Introduction

The problem of investigation of time delay systems has been exploited over many years. Time-delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, etc. The existence of pure time-delay, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability.

During the last four decades, the problem of stability analysis of time delay systems has received considerable attention and many papers dealing with this problem have appeared [1]. In the literature, various stability analysis techniques have been utilized to derive stability criteria for asymptotic stability of the time delay systems by many researchers [2]-[6]. The developed stability criteria are classified often into two categories according to their dependence on the size of the delay: delay-dependent and delay-independent stability criteria [7]. It has been shown that delay dependent stability conditions that take into account the size of delays, are generally less conservative than delay-independent ones which do not include any information on the size of delays.

Further, the delay-dependent stability conditions can be classified into two classes: frequency-domain (which are suitable for systems with a small number of heterogeneous delays) and time-domain approaches (for systems with a many heterogeneous delays).

In the first approach, we can include the two or several variable polynomials [8],[9] or the small gain theorem based approach [10].

In the second approach, we have the comparison principle based techniques [11] for functional differential equations [12]-[14] and respectively the Lyapunov stability approach with the Krasovskii and Razumikhin based methods [1],[15] The stability problem is thus reduced to one of finding solutions to Lyapunov [5] or Riccati equations [12], solving linear matrix inequalities (LMIs) [16],[17] or analyzing eigenvalue distribution of appropriate finite-dimensional matrices [18] or matrix pencils [10] . For further remarks on the methods see also the guided tours proposed by [13], [19]-[22].

In this paper we used of results [23],[24] for discrete-time delay systems. This is mainly due to the fact that such systems can be transformed into augmented high dimensional systems (equivalent systems) without delay.

The remainder of this paper is organized as follows. In Section 2, the problem statement and some necessary preliminaries are given. In Section 3, we proposed method for stability this systems. Numerical simulations are provided in Section 4. Finally, some concluding remarks are given in Section 5.

2. Problem statement

Consider a controllable linear time-invariant system with time-delay defined by the state equation

$$x(k+1) = \sum_{i=0}^{p} A_i\, x(k-i) + \sum_{i=0}^{q} B_i\, u(k-i), \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control input and the matrices A_i and B_i are real constant matrices of dimensions $n \times n$ and $n \times m$, respectively, with $rank(B_i) = m$.

By definition state vector such as

$$X_1(k+1) = \begin{pmatrix} x(k+1) \\ x(k) \\ \vdots \\ x(k-(p-1)) \\ u(k) \\ u(k-1) \\ \vdots \\ u(k-(q-1)) \end{pmatrix} \tag{2}$$

The system (1) with p delays in state and q delays in input vector can be rewritten as a standard system

$X_1(k+1) = AX(k) + Bu(k)$ (3)

Where

$$A = \begin{pmatrix} A_0 & A_1 & \cdots & A_{p-1} & A_p & B_1 & B_2 & \cdots & B_{q-1} & B_q \\ I_n & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & I_n & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & I_n & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & I_m & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & I_m & \cdots & 0 & 0 \\ 0 & 0 & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & I_m & 0 \end{pmatrix}, B = \begin{pmatrix} B_0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ I_m \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{4}$$

and $X_1 \in R^{\tilde{n}}, u \in R^m$, $\tilde{n} = n(p+1) + mq$.

We define control law as

$$u(k) = FX_1(k), \tag{5}$$

Where F is a feedback gain. Therefore, the system (1) changes to a standard closed-loop system

$$X_1(k+1) = (A+BF)X_1(k). \tag{6}$$

In this paper we determined the state feedback matrix F such that the eigenvalues of the closed-loop system $\Gamma = A + BF$ lie in the self-conjugate eigenvalue spectrum $\Lambda = \{\lambda_1, \lambda_2, \cdots \lambda_{\tilde{n}}\}$.

Karbassi and Bell [25]-[26] have introduced an algorithm obtaining an explicit parametric controller matrix F by performing three successive transformations T,S and R which transforms the controllable pair (A,B) into standard echelon form, primary vector companion form and parametric vector companion form, respectively. Let F represent the primary feedback matrix which assigns the desired set of eigenvalues to the closed-loop system.

3. Main results

Consider the state transformation

$$X_1(k) = T\tilde{X}_1(k), \tag{7}$$

where T can be obtained by elementary similarity operations as described in [26]. By replace (7) in equation (3) we have

$$\tilde{X}_1(k+1) = T^{-1}AT\,\tilde{X}_1(k) + T^{-1}B\tilde{u}(k), \tag{8}$$

In this way, $\tilde{A} = T^{-1}AT$ and $\tilde{B} = T^{-1}B$ are in a compact canonical form known as vector companion form:

$$\tilde{A} = \begin{bmatrix} G_0 \\ I_{\tilde{n}-m} \quad , \quad 0_{\tilde{n}-m,m} \end{bmatrix}, \qquad \tilde{B} = \begin{bmatrix} B_0 \\ 0_{\tilde{n}-m,m} \end{bmatrix}. \tag{9}$$