

A Brief Introduction on Prof. Rong's Mathematical Work

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In honor of Professor Xiaochun Rong on his seventieth birthday

Xiaochun Rong was born on June 17th, 1954. He graduated from Beijing Normal College (now Capital Normal University) in 1982, and received his Ph.D. from Stony Brook University in 1990. Rong was a Sloan Research Fellow in 1996-1998, and an NSF research grant awardee in 1994-2013. He has been a distinguished professor at Rutgers University in New Brunswick since 2008. Rong was a 2002 ICM invited speaker, and a 2017 Fellow of the American Mathematical Society.

Professor Rong is internationally recognized as a leading figure in Riemannian geometry for decades. Rong has made several seminal contributions to metric Riemannian geometry among them, a major part has been in collapsed Riemannian manifolds of various curvature bounds.

In particular, Rong proved that the fundamental group of a manifold with positive sectional curvature and symmetry rank one or with positively pinched curvature is virtually cyclic. Fang-Rong (independently, Petrunin-Tuschmann) proved a diffeomorphism finiteness for two-connected compact manifolds of bounded curvature and diameter. Rong solved several difficult conjectures in low dimensions, such as the Cheeger-Gromov Rationality conjectures on limiting eta-invariants and geometric signature and the Gromov's Gap conjecture on minimal volume. Cheeger-Rong proved a weak version of Gromov's Gap conjecture on minimal volume. Cao-Cheeger-Rong solved Buyalo's conjecture on collapsed manifolds of bounded non-positive sectional curvature. Rong-Zhang solved the Candelis-de-Laso Connectedness conjecture in Calabi-Yau manifolds. Chen-Rong-Xu proved a quantitative version of Maximal volume entropy rigidity of Laddrapier-Wang.

Rong has made contributions in areas including: manifolds of positive sectional curvature and large symmetry, rigidities/stability in Alexandrov geometry, and collapsed

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manifolds with local Ricci bounded covering geometry which was proposed by Rong. Fang-Rong gave a homeomorphism classification for compact 5-manifolds with positive sectional curvature and symmetry of rank two, and in collaboration (Fang, Su, Wang) he obtained homeomorphism classifications for manifolds with positive sectional curvature and various large symmetry ranks. In collaboration (Li, Wang, Xu) he proved several rigidity/stability results, and Perel'man's Soul conjecture in Alexandrov geometry in dimension four, and in collaboration (Huang, Kong, Xu) he established a Ricci version of Gromov's almost flat manifolds. Recently many works on collapsed manifolds with local Ricci bounded covering geometry have sprung up motivated by recent work of Rong.

Rong's research work in the past 35 years has a deep and long lasting impact on many directions of Riemannian geometry. Many of his works were published in prestigious mathematical journals: *Adv. Math.*, *Ann. of Math.*, *Duke*, *GAF*, *Invent. Math.*, *JDG*, etc. Some of his results have been exposed in textbooks and monographs.

Professor Rong has been an influential educator. He has supervised 17 Ph.D students (12 students completed their Ph.D Dissertations in China), also a mentor for six postdocs. Since 2003 he has visited Capital Normal University during every summer and winter vacation, where he taught regular/mini graduate courses, organized weekly reading/working seminars, workshops for young mathematicians, and organized several international mathematics conferences.