Optimal Control of a Fractional-Order New Psychoactive Substance Model*

Xinxin Wang[1], Xiaoyun Wang [1,†]

Abstract In this paper, we develop a fractional-order model of new psychoactive substance (NPS) transmission. We first prove the non-negativity and boundedness of the model. Then, prevention function and treatment function are introduced into the model to establish the fractional-order optimal control model. The existence of the optimal control pair is proved, and the solution of the model is obtained. Finally, the numerical simulation of the optimal control model is carried out. The results show that the fractional model is helpful for us to analyze the NPS dynamics model more deeply. Through the analysis of three control strategies, the optimal control strategy is the combination of prevention and treatment measures. In addition, it is worth noting that this paper obtains a new insight that prevention is more effective than treatment in the early stage of NPS transmission, and the control strategy at this time is to invest in maximum prevention measures and moderate treatment measures. However, when the spread of NPS shows an epidemic trend, the influence of preventive measures is greatly reduced, and the control strategy at this time is to focus on increasing the investigation of addicts and improving the treatment rate of psychological addicts.

Keywords New psychoactive substances, fractional order model, optimal control, numerical simulation

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1. Introduction

Over the past decade, new psychoactive substances have become a global phenomenon [1]. According to the United Nations Office on Drugs and Crime (UNODC), new psychoactive substances (NPS) are defined as substances that are not controlled by international drug control conventions but are subject to abuse and pose a threat to public health. These substances, which are generally obtained by modifying the chemical structure of existing drugs, not only have narcotic, excitatory or hallucinogenic effects similar to those of listed drugs, but also can evade legal control. As a result, the production, trafficking and abuse of these substances are becoming increasingly serious [2]. As of December 2021, the total number of new psychoactive substances reported by national authorities and forensic laboratories

was 1,127, which is more than three times the 302 psychoactive substances under international control at the end of 2021 [3] (Fig.1).
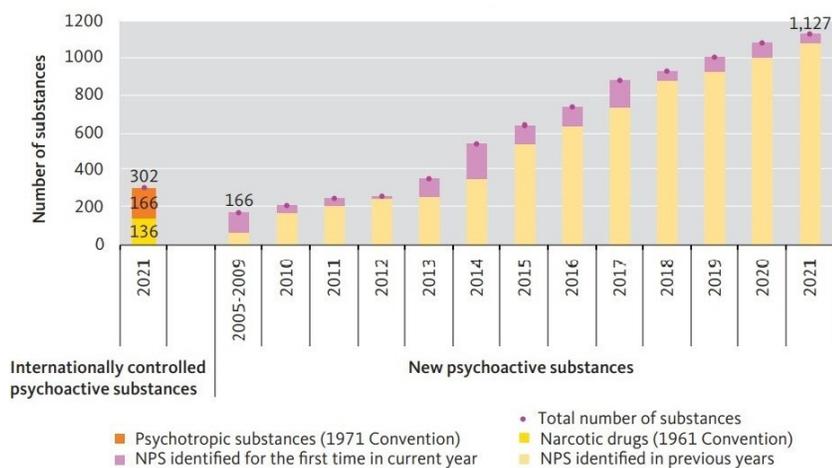

Figure 1. [3] The number of internationally controlled drugs in 2021, and the number of new psychoactive substances identified at the global level, 2005-2021 (cumulative figures)

These new psychoactive substances can not only seriously endanger the physical health of users, infecting them with various infectious diseases and even jeopardizing their lives, but they can also damage their nervous system, which in turn leads to psychotic symptoms [4,5]. More seriously, NPS directly affects the central nervous system, exhibiting greater dependence and infectiousness [6–8]. Studies have shown that the abuse of these substances leads to seven times more violent crimes than controlled substances [9]. In conclusion, new psychoactive substances pose a major threat to global public health [10].

Existing papers have shown that mathematical models can not only better explain drug transmission patterns but also provide predictive tools for the behavior of various types of drug users. White and Comiskey proposed a model in 2007 that divided drug users into two groups: those who are not in treatment (U_1) and those who are in treatment (U_2). Through the sensitivity analysis of the model, the stability of the system and the conditions for the existence of backward branches are proved. At the end of the article, a key result is that prevention is better than cure [11]. G. P. Samanta analyzed the heroin model with distributed time delay by improving the White and Comiskey heroin model [12]. In 2015, F. Nyabadza and J. Mushanyu studied two types of rehabilitation trends: inpatient rehabilitation and outpatient rehabilitation [13]. In 2017, Mingju Ma et al. proposed a new synthetic drug model that includes both psychological and physiological addicts. By analyzing the model, it can be concluded that controlling the spread of synthetic drugs is more effective than treating addicts [14]. Pengyan Liu established a model of synthetic drug transmission by dividing susceptible people into those with a history of drug use and those without [15].

Given limited resources, policy makers must consider minimizing overall costs while controlling the spread of disease. The optimal control system has been studied in many fields and achieved remarkable results. Neilan et al. (2010) proposed a SEIR epidemic model that applied optimal control theory to disease modeling.