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Stability and Bifurcation Analysis of the Nutrient-Microorganism Model*

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Abstract Stability analysis and bifurcation of the nutrient-microorganism model are presented in this paper. It is found that the model could experience the changes of the equilibrium points and the saddle-node, the Hopf and the codimension-2 Bogdanov-Takens bifurcations. The induced complex dynamics are also illustrated, by virtue of theory like the Sotomayor's theorem, the normal form and the universal unfolding. From the obtained results, some insights into interaction between the nutrient and the microorganism can be given. Further, numerical simulation is carried out to verify the theoretical results.

Keywords Nutrient-microorganism model, coexistence, Bogdanov-Takens bifurcation

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1. Introduction

The relationship in populations of nature has always been a hot topic in dynamics, in which the relationship between predator and prey has attracted researchers' attention. In the 1920s, a mathematical model [1, 2] between predator and prey was proposed by Lotka and Volterra. Since then, scientists have conducted more in-depth research on mathematical models of population relations. As a special predator-prey model, nutrient-microorganism model has also been considered extensively.

In 1996, Van Cappellen et al. [3] proposed that many microorganisms and chemical substances have taken their place in the circulation of substance. As we know, the interaction between microorganisms and nutrients is generally established through a decrease in nutrients and an increase in the number of microorganisms that feed on nutrients. For deeper exploration into the behavior of microorganisms and chemicals, Baurmann and Feudel established the model [4],

$$\begin{aligned} \frac{dB}{d\tau} &= \mu B^a N - mB, \\ \frac{dN}{d\tau} &= \phi(\hat{N} - N) - \varphi \frac{B}{B+L} BN, \end{aligned} \tag{1.1}$$

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where N represents the concentration of nutrients(mol/m^3), B is a bacterial population density that feeds on nutrients(kg biomass/m^3), a is the growth rate of bacteria, μ represents the conversion rate of biomass, m is the mortality rate of bacteria, φ is the capture rate of bacteria, and L is a semi saturated state constant. All parameters here are positive constants.

In the model (1.1), there are two ways to input nutrients. One is to input nutrients on the surface of the sediment, and the other is to input nutrients to the deeper sediment by bioirrigation technology. Because the depth of sediment affects the coverage rate of biological irrigation, the flux $\phi(\hat{N}-N)$ is used to deal with the diffusion term. Furthermore, it is assumed that only active portion of the bacteria is considered.

For model (1.1) with diffusion, Baurmann and Feudel [4] explore the turing patterns through a series of analysis and demonstration. In the same year, Baurmann et al. [5] proposed a new model by reconsidering the dormant bacteria and the activation of dormant bacteria into model (1.1). Then Turing instabilities and formation was found. Schmitz et al. [6] introduced a three species model in which nutrients are consumed by two competing populations of microbes in a marine sediment. Wetzel [7] investigated pattern formation of the benthic nutrient-microorganism model by Landau reductions and numerical methods and it is concluded that the system has an unstable state which means turing patterns start to exist for relatively small rates of food supply and ingestion. Furthermore, global bifurcation diagram for solutions over a bounded 2D domain is obtained. In Qian et al. [8], the local and the global bifurcations were considered in the diffusive nutrient-microorganism model by stability analysis, degree theory and bifurcation method. Moreover, the direction and the stability of the Hopf bifurcation was also obtained by considering the diffusive sediment model with no-flux boundary conditions in [9].

The delayed nutrient-microorganism model was put forwarded in [10] and its complex dynamical behavior was explored, including the codimension-2 bifurcation the Hopf-Hopf bifurcation. The resulting dynamical classification from bifurcation was also obtained by using the amplitude equations. Further results about the Hopf bifurcation in delayed nutrient-microorganism model with network structure were presented in [11] and dynamics of the diffusive nutrient-microorganism model with spatially heterogeneous environment was established in [12]. As further, the global existence and spatiotemporal pattern formation of the model with nutrient-taxis in the sediment [13] were given. The Hopf bifurcation of the diffusive nutrient-microorganism model with time delay was considered by applying the normal form theory and center manifold theorem [14]. For more detailed information about the nutrient-microorganism model, we refer to [15].

Although various dynamical results about the nutrient-microorganism model with the diffusion term, the delay and the taxis were reported, the bifurcation analysis for model (1.1) is also interesting and can be further explored in detail. There are few results in this area, then in this work qualitative analysis and bifurcation of model (1.1) will be carried out. To this end, make the following dimensionless transformation to system (1.1)

$$N=\frac{m}{\mu}v,\ \hat{N}=\frac{m}{\mu}\alpha,\ B=\frac{\phi}{\varphi}u,\ L=\frac{\phi}{\varphi}K,\ \tau=\frac{1}{\phi}t,\ \beta=\frac{m}{\phi},$$