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Bright-Dark Solitons, Kink Wave and Singular Periodic Wave Solutions for the Lonngren-Wave Equation

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Abstract In this article, exact solutions of the Lonngren-wave equation are investigated. Firstly, the equation is transformed into an ordinary differential equation by traveling wave transformation. Based on the homogeneous balance method, bright-solitons and singular periodic wave solutions of the equation are derived by applying the simple function expansion method and the Riccati equation method. Applying the $Exp(-\varphi(\varsigma))$ expansion method, we construct dark-solitons and kink wave solutions of the equation. Moreover, the 3-D, 2-D and density plots are drawn by choosing the appropriate parameters so that the properties of the solutions can be better studied. According to the Figures, the analysis of the dynamical behavior of the solutions is provided. This article enriches the diversity of the solutions of the equation.

Keywords Lonngren-wave equation, soliton solutions, kink wave solutions, singular periodic wave solutions

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1. Introduction

Researchers have devoted more and more attention to the study of nonlinear partial differential equations (NLPDEs), which have appeared in various fields, such as physical engineering and biomedical science [1]. Hence, finding the solutions of the equations is significant. There are many methods for solving NLPDEs, for instance, the Jacobi elliptic function method [2], the inverse (G'/G)-expansion method [3], the $Exp(-\varphi(\varsigma))$ expansion method [4,5], the improved F-expansion method [6], the sine-Gordon expansion approach [7], the Lie symmetry method [8–10], the Riccati equation method [11, 12], the extended simple equation method [13, 14], the homogeneous balance method [15,16], the Hirota bilinear transformation [17,18], the modified tanh method [19,20], the generalized unified method [21,22], etc [23,24].

Consider the following Lonngren-wave equation [25]

$$\left(v_{xx} - \alpha v + \beta v^2\right)_{tt} + v_{xx} = 0, \tag{1.1}$$

in which α and β are real constants. Many scholars have studied the properties of the Lonngren-wave equation. Eq. (1.1) describes the propagation of the electrical signal in the tunnel diode [26]. In [27], Durur and Hülya acquired the traveling

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wave solutions of Eq. (1.1) by applying the generalized exponential rational function method, and concluded that the velocity is an important factor affecting the wave diffraction by parameter assignment. Duran [28] investigated the effect of conductivity on the electrical signal propagation and distribution by taking values to the parameters in the solutions. Baskonus et al. [29] constructed the hyperbolic function solutions of Eq. (1.1) by the sine-Gordon expansion method. Yokuş [30] succeeded in deriving the soliton solutions by the auxiliary equation method, and investigated the effect of bright and dark-solitons on the charge distribution. Barman et al. [31] employed the generalized Kudryashov method to yield the wave structure under different parameters.

The equation can be used to explain the transmission of electrical signals in semiconducting materials and the storage of energy in charged circuits, which has great practical significance. In addition, the exact solutions of the equation can help explain the intrinsic motivation, such as the problem of the dispersion of electrical signals, especially in semiconductors. In this paper, three methods are used to obtain the corresponding bright-solitons, dark-solitons, and kink wave solutions in [31], but with different functional expressions. Furthermore, the singular periodic wave solutions of Eq.(1.1) are derived which are of great interest.

This article is organized into the following sections. In Section 2, Eq. (1.1) is converted into an ODE by the traveling wave transformation. In Section 3, bright-solitons solutions are yielded by utilizing the simple equation expansion method. In Section 4, based on the $Exp(-\varphi(\varsigma))$ expansion method, dark-solitons, singular periodic wave and kink wave solutions are constructed. In Section 5, applying the Riccati equation method, singular periodic wave and bright-solitons solutions of Eq. (1.1) are derived. A brief discussion of the obtained graphs is given in Section 6. Section 7 gives the remark and comparisons. Finally, the results of the study are summarized in Section 8.

2. Preliminary

The key steps of the extended simple equation method, the $Exp(-\varphi(\varsigma))$ expansion method and the Riccati equation method are given. Applying the above three methods, we obtain bright-dark solitons, singular periodic wave and kink wave solutions.

Assume that the NLPDE takes the following form

$$\Psi\left(v, v_x, v_t, v_{xx}, v_{tt}, \cdots\right) = 0, \tag{2.1}$$

in which Ψ is a polynomial function of $v = v(x,t)$ and its partial derivatives.

Consider the following traveling wave transformation

$$v\left(x,t\right) = V\left(\varsigma\right), \varsigma = kx - wt, \tag{2.2}$$

in which k, w are constants. Putting (2.2) into Eq. (2.1), we have

$$\Phi\left(V, V', V'', \cdots\right) = 0, \tag{2.3}$$

where the superscript is denoted as the derivative with respect to ς.

Eq. (1.1) can be rewritten as the following ODE

$$w^2k^2V^{(4)} - \alpha w^2V'' + 2\beta w^2V'' + 2\beta w^2VV'' + k^2V'' = 0. \tag{2.4}$$