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Exact Solutions of a (4+1)-Dimensional Boiti-Leon-Manna-Pempinelli (BLMP) Equation

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Abstract This paper focuses on constructing the exact solutions of a (4+1)-dimensional BLMP equation via the two variables $(G'/G, 1/G)$-expansion method and extended generalized Riccati equation mapping method. Firstly, the main ideas of the methods are described. Then, the methods are applied to the equation to derive the exact solutions including singular, kink (or anti-kink) and periodic solutions. Finally, the 3D plots of some exact solutions are observed graphically and intuitively by assigning the values of unknown parameters. The results prove that the methods are powerful, enriching the diversity of forms of exact solutions.

Keywords (4+1)-dimensional BLMP equation, exact solution, the two variables $(G'/G, 1/G)$-expansion method, the extended generalized Riccati equation mapping method

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1. Introduction

Nonlinear evolution equations (NLEEs) are frequently adopted to explicate complicated physical phenomena in numerous fields such as mathematical physics, chaos, quantum field theory, plasma physics, oceanography, etc. The completely integrable systems are claimed to be exactly solvable models among NLEEs. High-dimensional NLEEs are closer to actual natural phenomena and have more complex behavior. In order to study in depth the dynamic processes described by high-dimensional models, it is growing increasingly compelling to establish exact solutions that imply many physical properties of high-dimensional NLEEs. With the progress of technology and the efforts of researchers, extensive well-validated methods for solving fascinating nonlinear models are successively adapted, for instance, Darboux transformation [1], mETF method [2, 3], bifurcation analysis [4, 5], extended generalized Riccati equation mapping method [6–9], Hirota bilinear method [10], two variables $(G'/G, 1/G)$-expansion method [11–13], linear superposition method [14, 15], Lie symmetry method [16–18], etc. [19–21].

A (4+1)-dimensional BLMP equation [22] proposed by Xu and Wazwaz shall be studied, which reads

$$\begin{aligned} &\omega_t\left(\omega_y+\omega_z+\omega_s\right)+\sigma(\omega_y+\omega_z+\omega_s)_{xxx}\\ &+\mu\left(\omega_x\left(\omega_y+\omega_z+\omega_s\right)+\omega_{xx}\left(\omega_y+\omega_z+\omega_s\right)\right)=0, \end{aligned} \tag{1.1}$$

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where σ, μ are non-zero parameters and $\omega = \omega(x, y, z, s, t)$. Here, x, y, z, s represent spatial variables while t means time. Eq. (1.1) can be regarded as being evolved from the KdV equation in (4+1)-dimensions.

In recent years, breakthroughs have been gained in the research of Eq. (1.1). Among them, Painlevé properties held for Eq. (1.1) and Lax pair, bilinear Bäcklund transformation and infinite conservation laws were first considered by Xu and Wazwaz [22]. Hao [23] revealed block solitons, block kinks, periodic block solutions through the heuristic function method. Resonant multi-solitons and rational solutions were constructed by Kuo [24] and Hoessini et al. [25] via the linear superposition method, respectively. Raheel et al. [26] explored new solutions including periodic cross-kink wave solutions as well as interaction between kink solitary and rogue wave and secured these solutions. Moreover, the generalized exponential rational function method was utilized to derive explicit solitary wave solutions by Rasool et al. [27]. Motivated by them, we aim to explore exact solutions of Eq. (1.1) via the two variables $(G'/G, 1/G)$-expansion method and extended generalized Riccati equation mapping method in this paper.

It is worth pointing out that Eq. (1.1) can be converted into the following (2+1) and (3+1)-dimensional BLMP equations which catch people's eyes.

(i) When $\sigma = \mu = -1$ and $\omega = \omega(x, y, t)$, Eq. (1.1) is reduced to a (2+1)-dimensional BLMP equation presented by Boiti et al. [28] as follows

$$\omega_{yt} - \omega_{xxxy} - \omega_{xx}\omega_y - \omega_x\omega_{xy} = 0, \tag{1.2}$$

which is widely accepted to study incompressible liquids.

(ii) When $\sigma = 1, \mu = -3$ and $\omega = \omega(x, y, t)$, another form provided by Gilson et al. [29] is

$$\omega_{yt} + \omega_{xxxy} - 3\omega_{xx}\omega_y - 3\omega_x\omega_{xy} = 0, \tag{1.3}$$

whose integrability properties were verified by Luo [30] and various types of solutions were offered in [29–31].

(iii) When $\sigma = 1, \mu = -3$ and $\omega = \omega(x, y, z, t)$, a (3+1)-dimensional BLMP equation is formed as follows

$$\omega_{yt} + \omega_{zt} + \omega_{xxxy} + \omega_{xxxz} - 3\left(w_x\left(w_{xy} + w_{xz}\right) + w_{xx}\left(w_y + w_z\right)\right) = 0, \tag{1.4}$$

which describes the propagation of fluid. Numerous methods have been applied to construct lump-kink, multi-soliton, breather wave solutions, Painlevé analysis, Hirotas bilinear representation and so on [32–36].

The remaining plots are programmed as follows. Section 2 concisely provides the central thoughts of the two variables $(G'/G, 1/G)$-expansion method and extended generalized Riccati equation mapping method. The two methods are successively applied to Eq. (1.1) to summarize exact solutions including singular, kink (or anti-kink) and periodic solutions in Section 3. Section 4 performs some solutions graphically by using suitable parametric selections. Section 5 provides the discussion and comparisons. A summary is placed in Section 6.

2. Description of the methods

This section gives the brief steps of the methods considered. Discuss a NLPDE which is shown as

$$\Upsilon\left(\omega, \omega_t, \omega_{x_i}, \omega_{x_ix_j}, \omega_{x_ix_jx_\mu}, \cdots\right) = 0, \tag{2.1}$$