Journal of Nonlinear Modeling and Analysis http://jnma-online.com
Volume 6, Number 4, December 2024, 998–1021 DOI:10.12150/jnma.2024.998

Canard Phenomenon and Dynamics for a Slow-Fast Leslie-Gower Prey-Predator Model with Monod-Haldane Function Response

Xiao Wu[1] and Mingkang Ni[2,3,†]

Abstract The geometrical singular perturbation theory has been successfully applied in studying a large range of mathematical biological models with different time scales. In this paper, we use the geometrical singular perturbation theory to investigate a slow-fast Leslie-Gower prey-predator model with Monod-Haldane function response and get some interesting dynamical phenomena such as singular Hopf bifurcation, canard explosion phenomenon,r e laxation oscillation cycle, heteroclinic and homoclinic orbits and the coexis-t e nce of canard cycle and relaxation oscillation cycle.

Keywords Leslie-Gower prey-predator model, slow-fast system, canard explosion phenomenon, relaxation oscillation cycle, heteroclinic orbit, homoclinic orbit

MSC(2010) 34C37, 34C26, 34C07, 37G15.

1. Introduction

Predator-prey model is a typical theme in mathematical biology because of its wide application such as biological invasion of foreign species. In this paper, we mainly investigate the modified Leslie-Gower prey-predator model with Monod-Haldane function response [12, 21, 26] as follows

$$\begin{aligned} \frac{\mathrm{d}u}{\mathrm{d}t} &= u(a_1 - b_1 u) - \frac{c_1 uv}{u^2 + k_1}, \\ \frac{\mathrm{d}v}{\mathrm{d}t} &= v\left(a_2 - \frac{c_2 v}{u^2 + k_2}\right), \end{aligned} \tag{1.1}$$

where u and v separately represent the amount of prey and predators and all parameters are positive and have the following biological meanings: a_1 and a_2 are the natural growth rate of prey and predator which satisfy the assumption that the natural growth rate of prey a_1 is much larger than that of predators a_2; b_1 is the intraspecific competition rate of prey; c_1 measures the reduction of prey due

†the corresponding author.
Email address: XiaoWu2022@dhu.edu.cn(X. Wu), xiaovikdo@163.com(M. K. Ni)
1School of Mathematics and Statistics, Donghua University, Shanghai 200000, P. R. China
2School of Mathematical Sciences, East China Normal University, Shanghai 200000, P. R. China
3Shanghai Key Laboratory of Pure Mathematics and Mathematical Practice, Shanghai 200000, P. R. China

to predation; c_2 measures the reduction of predator because of the low density of prey; k_1 and k_2 represent the protection provided by the environment of prey and predators. Note that our assumption is reasonable because the lifespan of predators is very long and they may undergo many different generations of prey, such as hares and lynx, squirrels and coyotes.

For simplicity, using the rescaling transformation

$$\bar{t} = a_1 t, \quad \bar{u} = \frac{b_1 u}{a_1}, \quad \bar{v} = \frac{c_1 b_1^2 v}{a_1^3},$$
$$\epsilon = \frac{a_2}{a_1}, \quad \bar{c} = \frac{c_2 a_1}{c_1 a_2}, \quad \bar{k}_1 = \frac{k_1 b_1^2}{a_1^2}, \quad \bar{k}_2 = \frac{k_2 b_1^2}{a_1^2}$$

and dropping the bar notation, we rewrite system (1.1) as the following non-dimensional system

$$\begin{aligned} \frac{\mathrm{d}u}{\mathrm{d}t} &= u\left[1 - u - \frac{v}{u^2 + k_1}\right], \\ \frac{\mathrm{d}v}{\mathrm{d}t} &= \epsilon v\left(1 - \frac{cv}{u^2 + k_2}\right). \end{aligned} \tag{1.2}$$

Furthermore, with transformations

$$\tilde{t} = \int_0^t \frac{1}{(u^2 + k_1)(u^2 + k_2)} \mathrm{d}s$$

and $t = \tilde{t}$, system (1.2) can be rewritten as the following topological equivalent form

$$\begin{aligned} \frac{\mathrm{d}u}{\mathrm{d}t} &= u(u^2 + k_2)\left[(1 - u)(u^2 + k_1) - v\right] = f(u, v, \eta), \\ \frac{\mathrm{d}v}{\mathrm{d}t} &= \epsilon v(u^2 + k_1)(u^2 + k_2 - cv) = \epsilon g(u, v, \eta), \end{aligned} \tag{1.3}$$

where $\eta = (c, k_1, k_2)$ and $0 < \epsilon \ll 1$ under our assumption $a_2 \ll a_1$

It is clear that system (1.3) is a slow-fast system with one slow state variable v and one fast state variable u. Using the change of time scale $\tau = \epsilon t$, the system (1.3) can be transformed as

$$\begin{aligned} \epsilon\frac{\mathrm{d}u}{\mathrm{d}\tau} &= u(u^2 + k_2)\left[(1 - u)(u^2 + k_1) - v\right] = f(u, v, \eta), \\ \frac{\mathrm{d}v}{\mathrm{d}\tau} &= v(u^2 + k_1)(u^2 + k_2 - cv) = g(u, v, \eta). \end{aligned} \tag{1.4}$$

Compared with t, the time scale τ is the slow time scale. Hence, systems (1.3) and (1.4) are separately called the fast system and the slow system and their dynamics are equivalent if $0 < \epsilon \ll 1$.

Letting $\epsilon \to 0$ in systems (1.3) and (1.4), we get the degenerate system

$$\begin{aligned} 0 &= f(u, v, \eta), \\ \frac{\mathrm{d}v}{\mathrm{d}\tau} &= g(u, v, \eta), \end{aligned}$$