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Complex Dynamical Behaviors of a Leslie-Gower Predator-Prey Model with Herd Behavior*

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Abstract In this paper, we consider a Leslie-Gower predator-prey model with a square root functional response while prey forms a herd as a form of group defense. We show that the solution of the system is non-negative and bounded. By applying the blow-up technique, it can be deduced that the origin displays instability. Moreover, employing the proof-by-contradiction approach, we demonstrate that the unique equilibrium point can be globally asymptotically stable under certain conditions. The sufficient conditions for the occurrence, stability, and direction of Hopf bifurcation are obtained. We further explore the conditions for the existence and uniqueness of the limit cycle. Theoretical results are validated through numerical simulations. Thus, our findings reveal that herd behavior has an important impact on the Leslie-Gower prey-predator system.

Keywords Leslie-Gower predator-prey model, herd behavior, stability, Hopf bifurcation, limit cycle

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1. Introduction

The dynamical behavior between predator and prey plays a crucial role in the fields of biology, mathematics, and ecology. Establishing mathematical models is the preferred method for scholars to understand the dynamical behavior of the predator-prey system. With the advancement of ecology, the development of ecological models has become increasingly sophisticated. Among the models, the Leslie-Gower model [1] can be applied to describe the interaction and evolution process of two species in the ecosystem, illustrating that both prey and predator have their upper limit on growth rates. This interesting formulation for predator dynamics has been discussed by Leslie and Gower in [2] and by Pielou in [3]. The general form of the

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Leslie-Gower model is as follows

$$\begin{cases} \dot{x} = r_1 x(1 - \frac{x}{k}) - H(x)y, \\ \dot{y} = r_2 y(1 - \frac{y}{px}), \end{cases} \tag{1.1}$$

where $\frac{y}{px}$ in the second equation is called the Leslie-Gower term and p means the food quality of prey for conversion into predator's growth.

Since the model (1.1) was proposed, the functional response $H(x)$ describes the rate at which a predator consumes prey according to the density of prey and has been widely used in ecological models. However, in many real-world situations, predators may come together to form groups while searching for prey. This adds an additional layer of complexity that must be taken into account in the model. To address these complexities, functional responses that depend on both predator and prey characteristics have been developed. Integrating complex functional responses is crucial in comprehending the dynamics of predator-prey interactions and making precise predictions about ecosystem stability and biodiversity. There are two main categories for the exact forms of the functional response: prey-dependent and ratio-dependent (predator-dependent). This subject has been extensively studied by scholars of various disciplines [4–11]. Ding and Huang [8] classified the global dynamics of a ratio-dependent predator-prey model. The functional response takes the form $H(y/x)$, which depends on the ratio of predator and prey populations. Many results have been obtained in the study of limit cycles, including dissipativeness and permanence, local and global stability, periodic orbits, and various kinds of bifurcations. Limit cycles are a crucial area of research. It is a periodic oscillation in predator-prey systems that allows coexistence [12]. This dynamic mechanism has fascinated many researchers, not only in integer-order differential dynamical systems but also in fractional-order differential dynamical systems[13–15].

Group defense refers to the herd behavior of prey species gathering in groups to reduce the probability of being captured by predators. With appropriate assumptions about the form and type of prey's functional responses, the concept of group defense was considered and modeled in general terms [16]. Freedman and Wolkowicz [16] also considered Holling type IV functional response. If there is mutual interference among predators, it can improve their chances of survival. Ajraldi et al. [17] accounted for all types of interactions between populations, symbiosis, competition, and predator-prey interactions. They introduced a non-linear term, the square root of population density, which took into account the assumption that interactions occur along the boundaries of the populations. They assumed that x represents the total quantity of prey inhabiting a specific regular surface, such as circles, and then the predator at the group boundary consumed prey quantities proportional to $\sqrt{x}$. For example, this may be entirely appropriate for herbivores in a large savanna and their large predators. Based on their findings, the predator-prey model in [17] shows stable limit cycles and Hopf bifurcation, which are unique features compared to other predator-prey systems.

The integration of herd behavior of prey into a prey-predator system has also been studied in reference [18–28]. Braza [18] demonstrated the dynamical behavior of the origin. Xu et al. [19] proposed the conditions for periodic orbits and the existence and uniqueness of limit cycles. Bulai et al.[20] proposed replacing the general exponent α $(0 < \alpha < 1)$ with the exponent $\frac{1}{2}$. The stable solution attained by the populations is independent of the shape of the herd. In addition, it is