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On Controllability of Solution for Nonlinear Neutral Fuzzy Integro-Differential Equations

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Abstract This paper delves into an examination of the existence, uniqueness and controllability results concerning impulsive functional fuzzy nonlinear neutral integro-differential equations with non-local conditions. Additionally, we explore the fuzzy solution within the context of normal, convex, upper semi-continuous and compactly supported interval fuzzy numbers. Our findings are derived through the application of the Banach fixed point theorem. Also, we illustrate the result with an example.

Keywords Nonlinear neutral integro-differential equations, controllability, fuzzy solution, impulsive functional, nonlocal condition, fixed point

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1. Introduction

Many topics in studies relating to elasticity and viscosity have been modeled in several domains of physics and engineering due to their widespread applications. These problems are expressed using integral equations, differential equations, and integro-differential equations. In particular, neutral differential equations appear in many fields of applied mathematics, which is why they have received so much attention in recent decades [8, 13–16, 19, 20].

The significance of fuzzy differential equations within fuzzy analysis is supported by an extensive body of literature in the field, as referenced in [10, 11]. Over recent years, impulsive differential equations have emerged as a focal point of research. Additionally, incorporating a delay in the fuzzy model enables the exploration of more comprehensive situations, as discussed in [7, 17]. Fuzzy theory is a model used for uncertainty. Fuzzy integro-differential equations play the most important role in the analysis of phenomena with memory where imprecision is inherent. Chalishajar et al. [4] studied the existence of fuzzy solutions for nonlocal impulsive neutral functional differential equations. Najat et al. [12] studied the existence of impulsive fuzzy nonlinear integro-differential equations with the nonlocal condition by using the Leray-Schauder alternative fixed point theorem.

Controllability plays an important role in examining and suggesting control systems. It means the presence of a control function that directs the system's solution

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from its initial condition to the intended end state. Recently, there have been a few researchers who are interested in studying the controllability of fuzzy integro-differential systems. Chalishaja et al. [3] developed controllability for impulsive fuzzy neutral functional integro-differential equations using the Banach fixed point theorem. Kumar et al. [1] established the existence and total controllability result of a fuzzy delay differential equation with non-instantaneous impulses. Radhakrishan et al. [2] studied the controllability results for nonlinear impulsive fuzzy neutral integro-differential evolution systems.

In [3] and [5], the authors studied controllability for impulsive fuzzy neutral functional integro-differential equations of the type:

$$\begin{aligned}
&\frac{d}{d\kappa}[\rho(\kappa) - \mathfrak{z}(\kappa, \rho_\kappa)] = A\rho(\kappa) + \mathfrak{W}(\kappa, \rho_\kappa, \int_0^\kappa \mathcal{J}(\kappa, \eta, \rho_\eta) d\eta) + \varsigma(\kappa),\ \kappa \in [0, \mathcal{K}] = \mathcal{L},\\
&\Delta\rho(\kappa_n) = \mathcal{I}_n \rho(\kappa_n^-),\ \kappa \neq \kappa_n,\ n = 1, 2, \ldots, k,\\
&\rho(0) = \psi \in \mathbb{X}^d,
\end{aligned}$$

and

$$\begin{aligned}
&\frac{d}{d\kappa}[\rho(\kappa) - \mathfrak{z}(\kappa, \rho_\kappa)] = A\rho(\kappa) + \mathfrak{W}(\kappa, \rho_\kappa) + \varsigma(\kappa),\ \kappa \in [0, \mathcal{K}] = \mathcal{L},\\
&\Delta\rho(\kappa_n) = \mathcal{I}_n \rho(\kappa_n^-),\ \kappa \neq \kappa_n,\ n = 1, 2, \ldots, k,\\
&\psi(\kappa) = \rho(\kappa) + \mathfrak{h}(\rho_{\sigma_1}, \rho_{\sigma_2}, \ldots, \rho_{\sigma_q})(\kappa),\ \kappa \in [-\mathfrak{r}, 0],
\end{aligned}$$

respectively by using Banach fixed point theorem.

Motivated by the above mentioned work, here we consider the controllability of fuzzy solutions for impulsive functional nonlinear neutral integro-differential equations with nonlocal conditions of the following type:

$$\begin{aligned}
&\frac{d}{d\kappa}[\rho(\kappa) - \mathfrak{z}(\kappa, \rho_\kappa)] = A\rho(\kappa) + \mathfrak{W}(\kappa, \rho_\kappa, \int_0^\kappa \mathcal{J}(\kappa, \eta, \rho_\eta) d\eta) + \varsigma(\kappa),\ \kappa \in [0, \mathcal{K}] = \mathcal{L},\\
&\Delta\rho(\kappa_n) = \mathcal{I}_n \rho(\kappa_n^-),\ \kappa \neq \kappa_n,\ n = 1, 2, \ldots, k, \qquad (1.1)\\
&\psi(\kappa) = \rho(\kappa) + \mathfrak{h}(\rho_{\sigma_1}, \rho_{\sigma_2}, \ldots, \rho_{\sigma_q})(\kappa),\ \kappa \in [-\mathfrak{r}, 0],
\end{aligned}$$

where $A : \mathcal{L} \to \mathbb{X}^d$ is the fuzzy coefficient, $\mathbb{X}^d$ is the set of all convex, upper semicontinuous, and normal fuzzy numbers with bounded α-levels, $\mathfrak{W} : \mathcal{L} \times \mathbb{X}^d \times \mathbb{X}^d \to \mathbb{X}^d$, $\mathcal{J} : \mathcal{L} \times \mathcal{L} \times \mathbb{X}^d \to \mathbb{X}^d$ and $\mathfrak{h} : (C[-\mathfrak{r}, 0], \mathbb{X}^d)^q \to \mathbb{X}^d$ are regular fuzzy nonlinear functions, $\mathfrak{z} : \mathcal{L} \times C([-\mathfrak{r}, 0], \mathbb{X}^d) \to \mathbb{X}^d$ are continuous nonlinear functions, $\varsigma : \mathcal{L} \to \mathbb{X}^d$ is an admissible control function, $\psi : [-\mathfrak{r}, 0] \to \mathbb{X}^d$, and $\mathcal{I}_n \in C(\mathbb{X}^d, \mathbb{X}^d)$ are bounded functions. $\Delta\rho(\kappa_n) = \rho(\kappa_n^+) - \rho(\kappa_n^-)$, represents the left and right limits of $\rho(\kappa)$ at $\kappa = \kappa_n$, respectively, $n = 1, 2, \ldots, k$. $\rho(\kappa_n^+) = \lim_{\mathfrak{h} \to 0^+} \rho(\kappa_n + \mathfrak{h})$ and $\rho(\kappa_n^-) = \lim_{\mathfrak{h} \to 0^-} \rho(\kappa_n - \mathfrak{h})$. Moreover, $\rho_\kappa(.)$ represents the history where $\rho_\kappa = \rho(\kappa + \mathfrak{w})$; $\mathfrak{w} \in [-\mathfrak{r}, 0]$.

In this study, we generalize the results mentioned in [3] and [5], while also employing the Banach fixed point theorem to derive the existence, uniqueness, and controllability results for nonlinear impulsive fuzzy neutral integro-differential equations with nonlocal conditions.

This paper is organized as follows. We present preliminaries in section 2. We prove the existence, uniqueness and controllability of nonlinear fuzzy solution for