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Thermo-Electro-Elastic Friction Problem with Modified Signorini Contact Conditions

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Abstract The purpose of this paper is to investigate a frictional contact problem between a thermo-piezoelectric body and an obstacle (such as a foundation). The thermo-piezoelectric constitutive law is assumed to be nonlinear. Modified Signorini's contact conditions are used to describe the contact, and these are adjusted to account for temperature-dependent unilateral conditions, which are associated with a nonlocal Coulomb friction law. The problem is formulated as a coupled system of displacement field, electric potential, and temperature, which is solved using a variational approach. The existence of a weak solution is established through the utilization of elliptic quasi-variational inequalities, strongly monotone operators, and the fixed point method. Finally, an iterative method is suggested to solve the coupled system, and a convergence analysis is established under appropriate conditions.

Keywords Thermo-piezoelectric body, foundation, Signorini's modified contact conditions, Coulomb friction law, variational approach, elliptic quasi-variational inequalities, fixed point, iterative method

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1. Introduction

A notable set of challenges in engineering applications and technology pertains to the interaction between a deformable piezoelectric body and a conductive foundation. Practical instances of these challenges are prevalent in various sectors, including railways, automotive, civil engineering and aeronautics, among others. During these interactions, energy dissipation due to friction is a common phenomenon, resulting in the heating of the material. Moreover, specific piezoelectric structures display a pyroelectric effect, signifying their responsiveness to temperature variations, wherein exposure to different temperatures induces the generation of electric charge or voltage. Consequently, addressing a temperature load in a piezoelectric material requires a thorough consideration of interconnected thermo-electro-mechanical

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fields.

When studying friction contact models, the coupling of piezoelectric and thermal effects can create complications. The complex physical mechanisms of forces and heat on the contact interfaces are responsible for frictional effects and heating at the contact area. The dissipation of energy or production of heat can lead to dilation, which may increase the pressure field and modify the contact conditions, leading to instability. A brief history of the classical theory of thermo-piezoelectricity has been introduced by Mindlin in [1], while the governing equations and physical laws for thermo-piezoelectric materials have been explored by Nowacki in [2]. Primarily, owing to the intrinsic coupling between mechanical, electrical and thermal fields, significant research efforts have been devoted to addressing contact problems involving both piezoelectric and thermo-piezoelectric materials. The literature on electro-mechanical and thermo-electro-mechanical properties of piezoelectric materials is very interesting and extensive. We refer to [3–16] and the references therein.

This paper focuses on a new mathematical model, different from existing works, that describes the static contact with friction between a thermo-piezoelectric body and a thermally conductive foundation. Unlike previous references, this model differs in the way it models the contact, energy equation, thermal, frictional, and conductivity conditions. The contact is modeled using a modified Signorini's condition (refer to [17, Chapter 3, p. 147] for the linear elasticity case) and a version of Coulomb's friction law with a slip-dependent coefficient of friction. The heat flux is assumed to be unilateral from the foundation to the body, so the body temperature does not exceed the temperature of the foundation on the contact part. The model demonstrates strong coupling not only in the constitutive relations but also in the equilibrium equations and boundary conditions at the contact surface. From a mathematical point of view, the resulting model is well-posed, and weak solvability is established under appropriate assumptions on the problem's data. The proof relies on an abstract result on elliptic quasi-variational inequalities and Banach's fixed point.

The article is structured as follows: Firstly, in Section 2 we present some preliminary notations, definitions, and formulas that are necessary for the rest of the paper. Then, in Section 3 we state the mechanical problem and outline the assumptions on the data, followed by the derivation of a variational model. Section 4 is devoted to proving an existence and uniqueness result, utilizing quasi-variational inequalities and Banach's fixed point theorem. Lastly, in Section 5 we introduce an iterative approach for solving the resulting variational coupled system, which converges given certain conditions.

2. Notation and preliminaries

In this section, we will provide fundamental definitions, notations, and preliminary results that will be used in the subsequent sections. For further details, we refer the reader to references [19–21]. To this end, the summation convention over repeated indices is used, and all indices take values in $1, \ldots, d$. We denote by $\mathbb{S}^d$ the space of second order symmetric tensors on $\mathbb{R}^d$. We define the inner products and the corresponding norms on $\mathbb{R}^d$ and $\mathbb{S}^d$, that is $\forall u,\ v \in \mathbb{R}^d$ and $\forall \sigma,\ \tau \in \mathbb{S}^d$.

$$u \cdot v = u_i v_i, \quad \|v\| = (v \cdot v)^{\frac{1}{2}} \quad \text{and} \quad \sigma \cdot \tau = \sigma_{ij}\tau_{ij}, \quad \|\tau\| = (\tau \cdot \tau)^{\frac{1}{2}}.$$