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Analytical Method of Nitrogen Uptake Model for Plant Roots*

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Abstract The nitrogen uptake model for plant roots is an advection-diffusion equation subject to double Robin boundary conditions in Cartesian coordinates and its analytical method is expected to accurately estimate the quantity of nutrient uptake and fertilization. Firstly, the Michaelis-Menten (MM) kinetics function in the left boundary condition is changed into a function of time by numerical fitting and the nonlinear left Robin boundary condition then becomes a linear one in order to use traditional analytical methods. Based on the eigenfunction expansion method originally built by Golz and Dorroh, the nitrogen uptake model is homogenized and its eigenvalues are obtained from the Sturm-Liouville problem. Because the convergence of this eigenfunction expansion method is slow around the left boundary, i.e., root surface, we additionally consider the Laplace transform to solve the nitrogen uptake model. However, the solution after Laplace transform involves composite functions and numerical inverse Laplace transforms are introduced to obtain the final solutions. The analytical and numerical solutions show that the nitrogen concentration profiles along the distance from the root surface are convex upward and almost horizontal in the middle part with large gradients at both ends. The numerical simulation demonstrates that the eigenfunction expansion method can reach a satisfactory accuracy and the Laplace transform method with Stehfest inversion has higher calculation efficiency.

Keywords Nitrogen uptake, advection-diffusion equation, Robin boundaries, Golz and Dorroh's method, numerical inverse Laplace transform

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1. Introduction

Sufficient nutrient supply is important for the growth and harvest of crops [1]. Nitrogen is one of the most important nutrients for plants, and the main form of nitrogen absorbed by plant roots is inorganic [2]. On the contrary, excess nitrogen supply is detrimental to crops and wreaks havoc to N farmland [3, 4]. Hence, a

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thorough understanding of the nutrient absorption mechanism is of economic and scientific significance for the efficient use of chemical fertilizers and the increase of crop yields.

The movement of solutes from the surrounding soil into the roots can be effectively described by the advection-diffusion model (ADM) [5, 6, 7, 8, 9, 10]. Nutrient models of most plants took the root geometry into account and were built in cylindrical coordinates, otherwise in Cartesian coordinates, e.g., phytoplankton [11, 12, 13, 14]. Besides, most nutrient uptake models use Dirichlet or Neumann boundary conditions rather than Robin boundary condition which is in flux form and abided by mass conservation [15, 16, 17]. Therefore, it is still expected to build the analytical method for nutrient uptake models with double Robin boundary conditions in Cartesian coordinates.

Nutrient uptake models belong to the parabolic problem, more precisely to the advection-dispersion problem (ADP). Classical methods for solving one-dimensional ADE includes integral transforms, Green's function, variable separation, homotopy analysis, etc. [18, 19, 20, 21, 22, 23, 24]. Most analytical methods focused on ADE subject to the first or second type of boundary conditions [25, 26, 27, 28, 29]. There are fewer systematical results for ADE subject to double Robin boundary conditions in finite domain because of the mathematical and computational difficulty. Some researchers used the eigenfunction expansion method, and generalized integral transform technique to obtain the analytical solution of ADE with Robin boundary condition in finite domain [30, 31, 32, 33, 34, 35]. The nitrogen uptake model in this paper is an ADE subject to double Robin boundary conditions, but the right-hand side of left boundary condition is the Michaelis-Menten function of dependent variable. Golz and Dorroh [31] originally built an analytical method for the convection-diffusion equation with double Robin boundary conditions in Cartesian coordinates, which has been cited in [36, 37, 38], but none of them fully accomplished the application of this method. The nutrient uptake models conform to the general form proposed by Golz and Dorroh [31]. However, we still need to cope with the Michaelis-Menten function and the calculation. Because of the difficulty of this method, we will attempt to build another method with Laplace and inverse Laplace transforms for broader interest. [39, 40, 41].

The paper is organized as follows. Analytical methods are built to solve the nitrogen uptake model in Section 2. Analytical solutions and simulations are compared in Section 3. Finally, the conclusion of this paper is given in Section 4, and the numerical scheme is given respectively in Appendix A.

2. Model and analytical methods

2.1. Nitrogen uptake model

McMurtrie and Näsholm built a nitrogen uptake model that simulated the balance between the supply of plant available nitrogen and losses associated with its uptake by plant roots, soil microbes and other mechanisms [9]. It takes the forms in Cartesian coordinates as

$$b\frac{\partial C_N}{\partial t} = Db\frac{\partial^2 C_N}{\partial x^2} + v_0\frac{\partial C_N}{\partial x} - mC_N + S_N, \qquad 0 \le x \le l, \quad t \ge 0, \tag{2.1}$$

$$v_0 C_N(0,t) + Db\frac{\partial C_N(0,t)}{\partial x} = \frac{j_{rmax} k_N C_{s0}}{k_N C_{s0} + j_{rmax}}, \qquad t \ge 0, \tag{2.2}$$