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On Strong Form Of $\beta^* - I-$ Open Sets via Ideals Topological Spaces

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Abstract The concept of *strong* $\beta^* - I-$ open sets in ideal topological spaces is investigated and some of their properties are obtained, also we study these in relation to some other types of sets. Furthermore, by using the new notion, we define the strong $\beta^* - I-$ interior and strong $\beta^* - I-$ closure operators.

Keywords Local functions, ideal topological spaces, strong $\beta^* - I-$ open sets, strong $\beta^* - I-$ closed sets

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1. Introduction

Kuratowski established the fundamental concept of ideal topological spaces [23]. Later vaidyanathaswamy [30] studied the concept in point set topology. Hamlett and Janković [20] discovered a new topology τ^* that is finer than τ and uses previous ones. They also created the concept of ideal topological spaces and introduced a new Kuratowski closure operator cl^*. The contributions of Hamlett and Janković in ideal topological spaces started the generalization of several important features in general topology via topological ideal [21]. They also established the use of topological ideals in the extension of topological notions by introducing the concept of $I-$ open sets [22]. Hatir and Noiri introduced the ideas of $\alpha - I-$ open, $semi - I-$ open and $\beta - I-$ open sets in ideal topological spaces [18]. Hatir and Keskin introduced the idea of strong $\beta - I-$ open sets [16]. Ekici introduced the concepts of $\beta^* - I-$ open sets [10]. Aqeel and Bin Kuddah ([6], [5]) presented the concepts of $S.S^* - I-$ open sets and $S.P^* - I-$ open sets. Recently, the ideal topological spaces have evolved through paactical research that has studied many new concepts, including [7,13,27]. We define strong $\beta^* - I-$ open sets and strong $\beta^* - I-$ closed sets in this article. Several traits and qualities are investigated.

2. Preliminaries

In this section, we summarize the definitions and results that are needed in the sequel. By a space, we always mean a topological space (X, τ) with no separation properties assumed. If $A \subset X$, then $cl(A)$ and $int(A)$ denote the closure and interior

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of A in (X,τ), respectively. An ideal I in a topological space (X,τ) is a nonempty collection of subsets of X that satisfies the following two conditions [23]:

(i) If $A \in I$ and $B \subset A$, then $B \in I$.

(ii) If $A \in I$ and $B \in I$, then $A \cup B \in I$.

Let (X,τ) be a topological space, and I be an ideal on X. An ideal topological space is a topological space (X,τ) with an ideal I on X and it is denoted by (X,τ,I). For a subset $A \subset X$, $A^*(I,\tau) = \{x \in X : U \cap A \notin I$ for each neighborhood U of $x\}$ is called the local function of A with respect to I and τ [30]. It is obvious that $(.)^* : p(X) \to p(X)$ is a set operator. Throughout this paper, we use A^* instead of $A^*(I,\tau)$. $cl^*(A)$ and $int^*(A)$ denote the closure and interior of A in (X,τ^*) respectively. In [20], Note $cl^*(A) = A \cup A^*$ defines a Kuratowski operator for a topology τ^*, finer than τ. We start with recalling some lemmas and definitions that are necessary for this study in the sequel.

Among the results published in [1,2,4–6,10,12,14–16,18,19,22,24,25,28,29] we mention the following results in the form of Definition 2.1.

Definition 2.1. A subset A of an ideal topological space (X,τ,I) is called:

(i) pre $-$ open if $A \subset int(cl(A))$;

(ii) strong $pre^* - I-$ open($S.P^* - I-$ open) if $A \subset int^*(cl^*(A))$;

(iii) $\alpha - I-$ open if $A \subset int(cl^*(int(A)))$;

(iv) strong $\alpha^* - I-$ open($S.\alpha^* - I-$ open) if $A \subset int^*(cl^*(int^*(A)))$;

(v) $semi^* - I-$ open if $A \subset cl(int^*(A))$;

(vi) strong $semi^* - I-$ open($S.S^* - I-$ open) if $A \subset cl^*(int^*(A))$;

(vii) $\beta-$ open if $A \subset cl(int(cl(A)))$;

(viii) $\beta - I-$ open if $A \subset cl(int(cl^*(A)))$;

(ix) $\beta^* - I-$ open if $A \subset cl(int^*(cl(A)))$;

(x) strong $\beta - I-$ open($S.\beta - I-$ open) if $A \subset cl^*(int(cl^*(A)))$;

(xi) $b - I-$ open if $A \subset cl^*(int(A)) \cup int(cl^*(A))$;

(xii) weakly semi $-I-$ open if $A \subset cl^*(int(cl(A)))$;

(xiii) $*-$ dense in itself if $A \subset A^*$;

(xiv) $*-$ perfect if $A = A^*$;

(xv) $I-$ open if $A \subset int(A^*)$;

(xvi) Almost strong $-I-$ open if $A \subset cl^*(int(A^*))$;

(xvii) Regular $-$ open if $A = int(cl(A))$;

(xviii) $t-$ set if $int(A) = int(cl(A))$;

(xix) $t - I-$ set if $int(A) = int(cl^*(A))$;

(xx) $\delta - I-$ open if $int(cl^*(A)) \subset cl^*(int(A))$.

The complement of pre $-$ open ((resp. strong $pre^* - I-$ open, $\alpha - I-$ open,...)) sets is called pre $-$ closed ((resp. strong $pre^* - I-$ closed, $\alpha - I-$ closed,...)) sets.