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Square-Mean Pseudo S-Asymptotically (ω, c)-Periodic Mild Solutions to Some Stochastic Fractional Evolution Systems*

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Abstract In this paper, we introduce the concept of square-mean pseudo S-asymptotically (ω, c)-periodic for stochastic processes and establish some composition and convolution theorems for such stochastic processes. In addition, we investigate the existence and uniqueness of square-mean pseudo S-asymptotically (ω, c)-periodic mild solutions to some stochastic fractional integrodifferential equations. We illustrate our main results with an application to stochastic Weyl fractional integrodifferential equations.

Keywords Stochastic processes, stochastic evolution equations, Brownian motion, pseudo S-asymptotically (ω, c)-periodic functions

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1. Introduction

Many publications have studied the problem of periodicity of stochastic and deterministic evolution equations, because of its importance for both pure and applied mathematics. Many real-world phenomena do not satisfy conditions of strict periodicity, which are often hard to meet. Over the recent decades, researchers have developed some generalized quasi-periodic functions, such as almost periodic functions, asymptotic periodic functions, asymptotic almost periodic functions, pseudo almost periodic functions and S-asymptotic periodic functions and so on, to better investigate and represent these periodic behaviours and their mathematical models. These types of functions are not exactly periodic, but posses some periodic characteristics. They are helpful for modeling complex systems that have fluctuations or perturbations. For more details on these subjects, see [7, 12, 13, 16, 19, 20, 25] and references therein.

The topic of quasi-periodicity is very attractive and interesting to researchers because it includes several and diverse fascinating untreated problems. Inspired by

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the well-known differential equation of Mathieu

$$\varphi''(\tau) + [a - 2q\cos(2t)]\varphi(\tau) = 0,$$

with solution verifying $\varphi(t+\omega) = c\varphi(t)$, $\omega \in \mathbb{R}$, $c \in \mathbb{C}$ that appear as simulations in different scenarios, such as the firmness of train rails with passing locomotives and the cyclic fluctuations in population growth, Alvarez, Gómez and Pinto [3] introduced the category of (ω, c)-periodic functions. Several authors have been interested in the theory, such as Abadias et al. [1], Mophou and N'Guérékata [17], Kéré et al. [15], Khalladi et al. [14]. On the other hand, some mathematicians have also examined how small changes can affect (ω, c)-periodic functions in abstract spaces. For example, Alvarez, Castillo and Pinto [4, 5] defined the concepts of (ω, c)-asymptotically periodic functions and (ω, c)-pseudo periodic functions in abstract spaces and applied them to the abstract Cauchy problem of first order and the Lasota-Wazewska model with unbounded and ergodic production of red cells. Recently, the concept of pseudo S-asymptotically (ω, c)-periodic was introduced by Chang et al. [8], which extends the S-asymptotically ω-periodic functions. A study on fundamental properties and applications of S-asymptotically ω-periodic functions can be found in [6, 9, 12, 13, 19, 20].

It cannot be overlooked that the presence of random noise emanating from natural sources frequently makes physical phenomena fluctuate or perturb. Hence, to get a more precise model, it is required to include some stochastic terms in the systems. The existence of quasi-periodic solutions for the stochastic evolution equations is very limited (see [10, 11, 18, 23, 26] and references therein). It is natural to study the stochastic versions of the deterministic concepts mentioned before. According to our understanding and search, there is no previous work on the idea of square-mean pseudo S-asymptotically (ω, c)-periodic for stochastic processes, which is the main reason for this research. This issue is interesting and new, and hence, the question of whether there exists a (pseudo) S-asymptotically (ω, c)-periodic mild solution in square-mean sense is still untreated for stochastic evolution systems. The primary novelties and major contributions of this paper are listed as follows :

(i) We introduce a new concept of square-mean S-asymptotically and pseudo S-asymptotically (ω, c)-periodic for stochastic processes.

(ii) We establish some completeness, composition and convolution theorems for such stochastic processes.

(iii) We also investigate the existence and uniqueness of square-mean pseudo S-asymptotically (ω, c)-periodic mild solutions to the following class of stochastic fractional evolution equations:

$$\partial_\tau^\alpha \phi(\tau) = A\phi(\tau) + \int_{-\infty}^{\tau} b(\tau - s)A\phi(s)ds + g(\tau, \phi(\tau)) + f(\tau, \phi(\tau)\,(dW(\tau)/d\tau), \tag{1.1}$$

where $\tau \in \mathbb{R}$, ∂_τ^α denotes the Weyl fractional derivative of order $\alpha > 0$, $A : D(A) \subseteq \mathbb{L}^2(\Omega,\mathbb{H}) \to \mathbb{L}^2(\Omega,\mathbb{H})$ is a closed linear operator on a complex separable Hilbert space $\mathbb{L}^2(\Omega,\mathbb{H})$ (where $\mathbb{L}^2(\Omega,\mathbb{H})$ is an appropriate function space specified in Section 2) and generate an α-resolvent family $\{\mathcal{R}_\alpha(\tau)\}_{\tau\geq 0}$ on $\mathbb{H}$. g, f are $\mathbb{H}$-valued appropriate functions to be define later. Here $(W(\tau))_{\tau\in\mathbb{R}}$ represents a two-sided and standard one-dimensional Brownian motion on $\mathbb{H}$.