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# Impact of Plankton Body Size on a Stochastic Plankton System with Lévy Jumps*

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**Abstract** In consideration of the important impact of plankton body size and Lévy noise on plankton system, a stochastic phytoplankton-zooplankton system with Lévy jump is proposed and investigated in this paper. Firstly, we prove that there is a unique global positive solution to the system by using Lyapunov function and Itǒ formula. Then, some thresholds which depend on plankton body size are given, and they determine the extinction and weak persistence in the mean of plankton populations. In addition, the sufficient conditions for the existence of a stationary distribution of the solution are given. Finally, some numerical simulations are introduced to support the main theoretical results and illustrate the impact of plankton body size and environmental noise on plankton populations.

**Keywords** Phytoplankton-zooplankton system, Lévy jump, Lyapunov function, Itǒ formula, stationary distribution

**MSC(2010)** 34D05, 60H10.

## 1. Introduction

As the most abundant life form in the aquatic ecosystem, phytoplankton absorbs nutrients from water and generates energy through photosynthesis to support the entire biological community, including fish and heterotrophic bacteria. However, phytoplankton may grow out of control, resulting in toxic or harmful effects on humans, fish, marine mammal under anoxic conditions [1]. These have stimulated many scholars to study the dynamics of phytoplankton blooms in many different ways, so as to explore the possible mechanisms underlying the occurrence or termination of these blooms. Hence, finding some key factors affecting the growth mechanisms of phytoplankton is currently of great interest.

Phytoplankton are a polyphyletic of single-cell primary producers commonly existing in aquatic ecosystems [2]. It is worth noting that the size of phytoplankton cells plays a major part in the metabolism and growth rate of phytoplankton. Further experimental studies showed that the growth rate, metabolic rate and nutrient uptake all depend on the size of phytoplankton cells [3]. In addition, the size of zooplankton is considered to be another prominent element that can significantly affect

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the growth of aquatic plankton, because the size of zooplankton body can change the grazing choices of phytoplankton and maintain a clear state in the aquatic ecosystem [4]. To sum up, the cell size of phytoplankton and the body size of zooplankton are two vital factors affecting the dynamic mechanism of phytoplankton growth in aquatic ecosystems. Because the dynamic mechanism of phytoplankton growth can be qualitatively and quantitatively described by mathematical models, mathematical models, as a powerful tool, have attracted increasing attention from biological mathematicians. However, there are few mathematical ecological models to study the effects of phytoplankton cell size or zooplankton body size or both on the dynamic mechanism of phytoplankton blooms in recent years [5]. For example, Zhao et al. successively studied a phytoplankton-zooplankton model [6–8], and they found that phytoplankton cell size or zooplankton body size has important effects on the spatiotemporal dynamics and growth dynamics of phytoplankton in the complex aquatic environments. This provides a very good idea for future research on phytoplankton-zooplankton models. As we all know, many toxin-producing phytoplankton (TPP) can release toxic chemicals into the aquatic environment, which can inhibit the growth of zooplankton and even kill them. Furthermore, the dynamics of plankton system can be affected by the release toxin.

In fact, in the real aquatic environment, the growth of plankton is inevitably be affected by environmental noise, such as photosynthetic effective radiation, nutrient availability, water temperature, light, acidity, etc., which are usually unpredictable. In this way, it is meaningful to incorporate the unpredictable environmental factors into the aquatic ecosystem, which can help us gain a deeper understanding of the real aquatic ecosystem. Of course, the intrinsic growth rate and mortality of the plankton are always disturbed by environmental noise.

In addition, population system may suffer sudden environmental perturbations, such as, tsunami, earthquakes, volcanoes, floods or hurricanes [9]. Scheffer et al. [10] pointed out that all ecosystems are exposed to gradual changes in climate, nutrient loading, habitat fragmentation or biotic exploitation. However, this smooth change can be interrupted by sudden drastic switches to a contrasting state. In order to model the physical environmental disturbance (occasional catastrophic shocks), it is reasonable to consider another environmental noise, namely the L$\acute{e}$vy jump noise, into the underlying population system. Bao et al. [9,11] did some interesting works in this field.

Based on the above research works, in this paper, we will consider the following stochastic phytoplankton-zooplankton model:

$$\begin{cases} \frac{\mathrm{d}P(t)}{\mathrm{d}t} = P(t^-)[r(x)(1-\frac{P(t^-)}{K}) - \alpha C(x,y)Z(t^-)]\mathrm{d}t + \sigma_1 P(t^-)\mathrm{d}B_1(t) \\ \qquad\qquad +P(t^-)\int_{\mathbb{Y}}\gamma_1(u)\widetilde{N}(\mathrm{d}t,\mathrm{d}u), \\ \frac{\mathrm{d}Z(t)}{\mathrm{d}t} = Z(t^-)[\beta C(x,y)P(t^-) - \mu - \frac{\theta P^2(t^-)}{m^2+P^2(t^-)}]\mathrm{d}t + \sigma_2 Z(t^-)\mathrm{d}B_2(t) \\ \qquad\qquad +Z(t^-)\int_{\mathbb{Y}}\gamma_2(u)\widetilde{N}(\mathrm{d}t,\mathrm{d}u), \end{cases} \tag{1.1}$$

where $P(t)$ is the density of toxin producing phytoplankton population and $Z(t)$ is the density of zooplankton population at time $t$. All parameters are nonnegative.$K$ is the environmental carrying capacity of TPP population; $\mu$ denotes the natural death rate of zooplankton; $\alpha$ is the rate of predation and $\beta$ is the conversion rate of zooplankton; $\theta$ is TPP toxin releasing rate; according to [12] ,the function $\frac{P^2(t)}{m^2+P^2(t)}$ describes the distribution of toxic substance which ultimately contributes to the